

Introduction to
**Modern
Navigation
Systems**

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To my Lord

with humility and love

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Preface

The goal of this text is to concisely present the mathematical blocks needed for implementing the main body of a strapped down Inertial Navigation System (INS) in a manner that provides a mental image of the contribution of each block and their interrelation. The text describes the makeup of each block and provides the derivation of its equations. Towards this objective, when the need for clarifying or justifying a certain idea arises, it is presented in an appendix so as not to interfere with the flow of the main ideas.

This treatment should benefit both the novice as well as a practitioner in the field. For a journeyman in the area of navigation, this book can be used to pinpoint the equations that are the basis of such a system, how they are developed and how they are implemented. Those with more experience may use this book as a quick reference guide.

What is navigation anyway? It is the ability to set the course of a ship to move between two desired locations. To do that the navigator must be able to know his location and set the velocity vector towards the desired destination. Thus the prime function of a navigation system is determining the craft's position and velocity.

We will be primarily concerned herein with a special type of navigation: inertial navigation. And why inertial navigation in particular? Inertial systems are self-contained: they are independent of weather conditions and are operable anywhere in seas, underwater, lands, tunnels, or in air. Short of a reliable source of power, they can work almost indefinitely.

If the Earth were flat, inertial navigation algorithms would have been a lot easier. Because navigation usually is on or close to Earth, a spherical body that rotates about itself, we will soon find ourselves entangled in discussing two different elements at the same time: developing the mathematical algorithms and describing the pertinent physics of the Earth.

We have devoted the first three chapters to introduce the mathematical foundation for developing the algorithms. These algorithms rely heavily on vector and matrix notations, and for that, vector and matrix properties are introduced in Chapter 1. On developing the equations, we will discover that our variables of interest are represented in different coordinate systems. Obviously this creates the need of moving from one coordinate system to another and thus this concept is discussed in Chapter 2. For further clarity, Chapter 3 introduces the most common approaches used in performing coordinate transformations. There we discuss these approaches and their relationships amongst one another.

We discuss the physical properties of the Earth in Chapter 4. At this point, armed with the mathematical tools and the geometrical properties of Earth, we develop the inertial navigation equations from first principles in Chapter 5. This yields a set of continuous time differential equations that should be solved to yield the navigation solution. Mechanizing and implementing these equations on a digital computer is introduced in Chapter 6. We discover one of the drawbacks of inertial navigation systems: unreliable vertical channel. This means that they cannot be relied on to provide altitude or vertical velocity.

Typically, the navigation system for aircraft is usually complemented with some sort of altimeter. Integrating the altimeter measurements with the INS is discussed therein. Our reliance on air data for aiding the navigation system does not end at this point: the INS cannot estimate wind speed. Chapter 7 is devoted to the discussion of air data computations and their use for computing rate of climb/descent and relative airspeed.

Using the legendary lines of longitude and latitude to locate a craft location on the surface of the Earth introduces a peculiar navigational

phenomenon: all longitude lines meet at the two polar points of the Earth. Solving navigation equations near these two points can prove to be mathematically cumbersome. Despite the rarity of these events, considerable attention has been given to avert the consequences of such an occurrence. The wander azimuth angle is one such classical technique and it is critiqued in Chapter 8. An innovative simple algorithm for navigating in the polar circle is introduced shortly thereafter.

Two problems remain to be solved. The first addresses the alignment problem. Simply stated, it is determining the initial conditions of the differential equations that were developed in Chapter 5. The second deals with the real life factors: no matter how expensive the inertial sensors are they have errors that must be estimated. Solutions to these problems depends on the inertial sensor level accuracy onboard the specific craft.

To elaborate, we may have noticed that implementing the INS equations not only provide the craft location and speed, but also its attitude and heading. Some applications use navigation grade sensors to estimate all the above parameters. But others utilize low-grade inexpensive sensors focused on estimating only the attitude and heading. In so doing, these applications – called Attitude and Heading Reference Systems (AHRS) – forgo estimating the location and velocity.

Chapter 9 addresses the alignment problem for navigation systems. In Chapter 10, we discuss the AHRS systems, introduce their pertinent alignment algorithms, and estimate sensor errors. Often, an AHRS complements its inertial system with magnetic detectors and utilizes mathematical algorithms to estimate the relatively large errors introduced by the inertial systems.

A system that utilizes navigation grade sensors could enhance its performance by using aids such as a Global Positioning System (GPS). Like the INS, the GPS provides position and velocity. These two, INS and GPS, when mathematically fused with a Kalman filter, can be used to estimate the inaccuracies due to the sensors of the former system. These equations are developed in Chapter 11.

This book could not be a reality without the help and support from many friends and colleagues. I am very grateful to A. Bekir and H. Aleem for reviewing the script and suggesting many improvements. I

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Contents

Preface	vii
Introduction	1
1. Vectors and Matrices	7
1.1 Introduction	7
1.2 Vector Inner Product	9
1.3 Vector Cross Products and Skew Symmetric Matrix Algebra	10
2. Coordinate Transformation between Orthonormal Frames	17
2.1 Introduction	17
2.2 Direction Cosine Matrices	18
2.3 The Direction Cosine Matrix is a Unitary Matrix	20
2.4 The Direction Cosine Matrix is a Transformation Matrix	21
2.5 DCM Fixed Axis	24
2.6 The Rotation Matrix	26
2.7 Inner and Outer Transformation Matrices	29
2.8 The Quaternion	32
3. Forms of the Transformation Matrix	35
3.1 Introduction	35
3.2 Simple Frame Rotations	36
3.3 Euler Angles	37
3.4 Rotation Vector	38
3.5 Quaternion	39
3.6 Simple Quaternions	43
3.7 Conversion between Forms	45
3.7.1 Conversion between DCM and Euler	45
3.7.2 Conversion between DCM and Quaternion	45
3.7.3 Conversion between Euler Angles and Quaternion	47

3.8	Dynamics of the Transformation Matrix	47
3.8.1	DCM Differential Equation	48
3.8.2	Quaternion Differential Equation	50
3.8.3	Rotation Vector Differential Equation	52
3.8.4	Euler Angles Differential Equation	55
4.	Earth and Navigation	58
4.1	Introduction	58
4.2	Earth, Geoid and Ellipsoid	59
4.3	Radii of Curvature	63
4.4	Earth, Inertial and Navigation Frames	65
4.5	Earth Rate	67
4.6	The Craft Rate ω_{en}^n	67
4.7	Solution of the DCM C_e^n	70
4.8	Gravitational and Gravity Fields	70
5.	The Inertial Navigation System Equations	75
5.1	Introduction	75
5.2	Body Frame of Reference	76
5.3	Inertial Sensors	77
5.3.1	The Accelerometer	77
5.3.2	The Rate Gyro	78
5.4	The Attitude Equation	78
5.5	The Navigation Equation	80
5.6	Navigation Equations Computational Flow Diagram	83
5.7	The Navigation Equation in Earth Frame	84
6.	Implementation	86
6.1	Introduction	86
6.2	The Rotation Vector Differential Equation	87
6.3	The Attitude Equation	92
6.4	The Craft Velocity Equation	95
6.5	The Craft Position Equation	99
6.6	The Vertical Channel	101
7.	Air Data Computer	104
7.1	Introduction	104
7.2	US Standard Atmosphere 1976	105
7.3	Pressure Altitude	107
7.4	Vertical Channel Parameter Estimation Using Inertial and Air Data	111
7.5	Density Altitude	116

7.6	Altitude (Descend /Climb) Rate	117
7.7	Air Speed	117
7.8	Indicated Air Speed (IAS)	119
8.	Polar Navigation	121
8.1	Introduction	121
8.2	The Wander Azimuth Navigation	123
8.3	Prospective of the Wander Azimuth Approach	126
8.4	Polar Circle Navigation Algorithm	128
8.5	Alternative Polar Circle Navigation Frame	132
9.	Alignment	136
9.1	Introduction	136
9.2	IMU Alignment	137
9.3	Alternative Algorithm for C_n^b	144
9.4	Estimation of the Accelerometer and Gyro Biases	149
9.5	Effects of Biases on Estimate of C_n^b	150
10.	Attitude and Heading Reference System	152
10.1	Introduction	152
10.2	Attitude Initialization	152
10.3	Heading Initialization	155
10.4	Gyro Drift Compensation	159
10.5	G Slaving	160
10.5.1	X-Gyro Bias	160
10.5.2	Y-Gyro Bias	162
10.5.3	Z-Gyro Bias	163
10.6	Alternative Approach for Gyro Drift Compensation	163
10.7	Maneuver Detector	165
10.7.1	Rate Gyro Threshold Selection	165
11.	GPS Aided Inertial System	167
11.1	Introduction	167
11.2	Navigation Frame Error Equation	168
11.2.1	Craft Rate Error $\delta\omega_{en}^n$	169
11.2.2	Earth Rate Error $\delta\omega_{ie}^n$	170
11.2.3	Position Errors	171
11.2.4	Attitude Error	173
11.2.5	Gravity Error	176
11.2.6	Velocity Error	177
11.2.7	Navigation Frame Error State Equation	179
11.2.8	Error Block Diagram	179

11.3 Earth Frame Error Equations	180
11.3.1 Attitude Error	181
11.3.2 Velocity Error	182
11.3.3 Position Error	183
11.3.4 Earth Frame Error State Equation	183
11.4 Inertial Sensors Error Models	183
11.5 The Global Positioning System	187
11.6 Mechanization of the INS/GPS Equations	191
Appendix A. The Vector Dot and Cross Products	194
Appendix B. Introduction to Quaternion Algebra	197
Appendix C. Simulink [®] Models	202
Appendix D. Ellipse Geometry	206
Appendix E. Vector Dynamics	213
Appendix F. Derivation of Air Speed Equations	219
Appendix G. DCM Error Algebra	222
Appendix H. Kalman Filtering	226
Index	237

Introduction

Where am I? Sound familiar? We travel to some destination then we realize we lost our way. We agonize upon it when we overshoot or undershoot our destination. Or when we take the wrong route or the wrong freeway exit and find ourselves in a foreign territory. Quite often these incidents end up without incident, but in few occurrences they could result in unhappy endings. This is not a modern day problem but is something that man has lived with since the beginning of time. People needed to migrate to different lands in search of food and water. They needed to travel to trade with other people living in different areas. In so doing they did not migrate or travel en masse; they probably sent scouts to explore the unknown territories who had to return to inform their tribes with their findings. The scouts must have used some landmarks or followed some terrains so they could return safely. But what would they have done when the land they traveled over had no distinguishing features. They must have established some sort of bearing that could guide them in their back and forth trips. Similar measures must have been used when sailing the open seas; a ship with an unknown bearing was a vessel of the doomed.

The priority of navigation with the ancients has not changed over several millenniums and is just as relevant as it is with today's field of modern navigation. It is centered upon one core, unshakeable theme: location, location, location. If you want to move from one place to another, the first thing you need to know is your location. Next you should be able to set the velocity vector of your transportation means towards the desired destination.

Navigation is derived of the Latin verb *navigare*, to sail, which also is derived of '*navis*' a ship. Navigation has been the science, or very much the art, of determining the position and the velocity of a craft, whether a car, a ship or an airplane. It is of little wonder that navigation become intertwined with sea and ships because it, by large means, evolved onboard of ships facing the enormous challenges of charting their courses. There are certain spots in water that are too shallow, rocky or turbulent that an experienced sailor would like to avert. Early travelers identified their locations and charted their routs by landmarks. When they got into water they, most probably, hugged very closely to the shoreline.

Navigation rose to higher level of sophistication when ancient Greeks realized that the Earth is spherical and when Eratosthenes of Alexandria measured its radius. They drew geographical maps superimposed on latitude and longitude quadrants.

Early on, people were aware that the sun and the stars can guide them when traveling long distances in the desert or in the sea. They knew that the altitude of Polaris, (the north star), indicates the latitude at which they locate. Near the equator (when Polaris is on the horizon and is difficult to see) or in daytime they had other means. They realized that at any point on Earth the sun attains its highest altitude at noon and this is when it points to the north and with a simple adjustment they can relate this altitude to the latitude at that point. Therefore, day or night, they could figure out the latitude. Navigators devised simple means for charting their routes: determine the initial latitude at the location they embark, sail north or south to the latitude of the desired destination and finally move east or west till they encounter their destination.

The latitude approach for navigation came into place because means for measuring the latitude were possible. Initially they came in the shape of the astrolabe, a simple apparatus, for measuring the altitude of the stars. The astrolabe depended on measuring the star altitude with respect to the horizon, but this meant poor measurements if the boat is rocking for bad weather or any reason. The astrolabe has evolved into the sextant, a more accurate tool [1].

Determining the latitude has solved one parameter of the navigation equation and it remained to determine the longitude. The magnetic

needle is discovered, but probably it was not of much help because it would not point to the true geographic north, nor did it maintain a constant error. The magnetic compass would be of great use when magnetic variations, the difference between magnetic north and the true north, were tabulated.

Crude means for measuring the ship speed became also available that come in the form of chip log [2]. The means took the shape of a relatively heavy slug is attached to a long rope of about 700 feet length, that was knotted every 47 feet and 3 inches. If the sailor wanted to measure the boat speed, he would toss the slug into the water and let the rope slip in his hands. He would wait for a 28 seconds during which he is counting the number of knots that slips between his fingers. The time was measured by nothing more than the legendary sandglass. Interestingly, this is why the ship speed is still currently measured in 'knots'.

Between the sextant, the magnetic compass and the ship speed, navigators were able to chart their ship courses to move to different places. It should be remembered that these computations were not accurate and possibly crude. A rolling ship would certainly prevent collecting accurate sextant measurements as the sextant must be steady and resting on a perfectly horizontal surface. Crude, may be, but good enough to go to places. With these tools Christopher Columbus set sail to the Americas and charted his way back home.

Columbus trip in the final years of the 15th century was a monumental epoch in navigation; it opened a new era for longer trips that lasted longer periods in uncharted waters to uncharted lands. Ships tended to become larger to carry more people and larger cargos to sail between the new lands and Europe. Older methods and navigation tools resulted in larger tragedies and needed to be improved.

It was not until the 18th century that navigation got a great boost when John Harrison invented and developed the 'chronometer' and perfected it to measure time with an error of less than .1 seconds per day. Recall earlier that sun at noon points to the north and that, in a way, points to local longitude. Suppose a navigator set sail from a point on the Greenwich line (i.e. 0 longitude) and purposely sets the clock at noon to 12:00PM. Few days later, the clock is found to read 11:00 at noontime,

the navigator would immediately know that the local longitude is 15 degrees east. This is because the Earth is girded with 360 longitude lines and since the Earth makes a complete circle every 24 hours, noon will differ by one hour every $(360/24)$ 15 degrees of longitude. With Harrison's chronometer, British naval officer and explorer James Cook was able in 1772-1775 to circumnavigate the Earth [3]. From information he gathered on his voyage, Cook completed many detailed charts of the world that completely changed the nature of navigation.

More accurate and much lighter sextants continued to evolve. However rough waters, which causes the ship to roll unsteadily, and fog, which causes the true horizon to be obscured, rendered the sextants very difficult to use under these conditions. Tools were needed to steady the sextant. Nothing but gyroscopic action that could do the job. Indeed it was a very fortunate moment when the roads of navigation and gyroscopes crossed and opened a new page in the history of navigation.

A gyroscope, basically, is a top that when rotates at very large rates, its upper surface would be horizontal and its axis of rotation and the Earth's axis make a plane that point to the north. What's more is that if the surface, on which the top spins, is disturbed the top will restore its phenomenon unless it is disturbed once more [4]. Amazing isn't it! It worth noting to mention that it was Foucault that named it 'gyroscope' in 1852. Not much time later when Admiral Fleuriais developed the gyroscopic sextant in 1885. But this is when the sextant reached its great year and started to give way to the gyro. Few factors has hastened the development of the new gyro: Larger ships made with iron alloys rendered the use of magnetic compasses very difficult, and temporal navigation and orientation tools – magnetic compasses and sextants – in submersibles were of little use.

The early days of the 20th century witnessed the dawn of the airborne aviation, and with it came the need for navigation tools on board of these flying machines. The legacy compass was still used to point direction. The new gyros, at the same time were being developed into the vertical and directional instruments to indicate the vertical and azimuth directions respectively. It turned out that the human sensing for the vertical direction while flying is very poor and hence the need for the vertical gyro to fly straight and level.

It all started by inventing and developing the 'Gyrocompass' – the practical instrument that indicates the direction to the north [5]. It performs well up to 80 degrees latitude. The gyrocompass was invented and developed by Dr. Hermann Anschuetz-Kaempfe in Kiel, Germany in 1908 [6]. “Compared with the conventional marine-type magnetic compass, the gyrocompass was far more accurate, being capable of indicating the north within a small fraction of one degree. It has no variation error” [7]. The gyrocompass was also developed in the USA by Elmer Sperry whose company has produced many versions. These gyrocompasses were installed on board commercial and navy ships.

The era of inertial navigation has started when gyros and accelerometers were used as a guidance tools in the German V2 rockets. With the end of the Second World War came a speedy activity for developing an inertial navigation system. This system comprises one triad of accelerometers and another triad of gyros, both are mounted on top of a platform called the stable element. The stable element is mounted on two or more orthogonal gimbals that allow it to have a complete three degrees of freedom orientation. With the gyros aid, the system is so designed that, the stable element will maintain its attitude with respect to the surface of the Earth. This attitude, as an example, is parallel to the local surface of the Earth and pointing to the north. With this arrangement, the accelerometer outputs are integrated once to give the craft velocity and integrated once more to give its location. Indeed it is a remarkable tool for navigation. Short of a reliable source of power, it can work almost indefinitely. With one of these systems the US Navy Nautilus navigated under ice to reach the North Pole in 1958.

This inertial navigation system is self-contained: it is independent of weather conditions and is operable anywhere – in seas, underwater, lands, tunnels, or in air. On the other hand it is very costly (in the order of \$100,000 in the 1970s), bulky and comprises many delicate components, sensitive to temperature variations and hence is very expensive to maintain.

In 1956, W. Newell patented the idea of the Strapdown Inertial Navigation System (INS). The patent described the implementation of an INS strapped down of its gimbals and literally fixing the inertial platform to the body of the craft. The thought is to trade the mechanical

orientation with an analytical one with use of onboard computer. True that inertial platform will not maintain its attitude with respect to the surface of Earth and as such the accelerations measured by the accelerometers can't be integrated to obtain the velocity and position of the vehicle. But on the other hand the attitude of the craft can be tracked accurately by the mounting gyros. This demanded the use of highly accurate gyros and the computer that can perform these intensive computations. At the time of the patent, lack of digital computers of reasonable size delayed the development of such system till the early 1970's. When they became available, a new page of modern navigation has begun.

It is the subject of this book to introduce the mathematical and physical concepts of the modern inertial navigation system. Also to formulate the equations that should be implemented on the computer to deduce the navigation solution.

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