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We may also refer to the online information, cf. App. 2, containing MATLAB applications.
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HANS PACEJKA

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