Finite Element Analysis (FEA) has been widely implemented by the automotive industry as a product design tool for design engineers who use it to reduce product development time and cost. This book serves as a guide for FEA users and addresses the specific needs of design engineers. It provides clear presentation of FEA fundamentals and implementation that will help practitioners to avoid misapplication of this tool. Easy to follow examples of FEA fundamentals are clearly presented and are ready for implementation during the product design process. The FEA is fully explored in this practical approach that includes:

- Understanding FEA basics
- Commonly used modeling techniques
- Commonly used types of FEA analyses
- Frequently made FEA errors and their effect on the quality of results
- Implementation of FEA in the design process
- Hands-on simple and informative exercises

This indispensable guide provides design engineers with proven methods to analyze their own work while it is still in the form of easily modifiable CAD models. Exercises provide examples for improving the FEA hands-on process to deliver quick turnaround times and prompt implementation.

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