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Road Vehicle Dynamics

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# Contents

Foreword.................................................................................................................................xvii

Preface......................................................................................................................................xix

Chapter 1 Introduction ............................................................................................................ 1

1.1 General .............................................................................................................................. 1

1.2 Vehicle System Classification......................................................................................... 2

1.3 Dynamic System.............................................................................................................. 3

1.4 Classification of Dynamic System Models..................................................................... 4

1.5 Constraints, Generalized Coordinates, and Degrees of Freedom.............................. 4

1.6 Discrete and Continuous Systems .................................................................................. 10

1.7 Vibration Analysis .......................................................................................................... 10

1.8 Elements of Vibrating Systems....................................................................................... 15

1.8.1 Spring Elements ........................................................................................................ 15

1.8.2 Potential Energy of Linear Springs .......................................................................... 18

1.8.3 Equivalent Springs .................................................................................................... 18

1.8.3.1 Springs in Parallel ............................................................................................ 19

1.8.3.2 Springs in Series ............................................................................................. 20

1.8.4 Mass or Inertia Elements.......................................................................................... 25

1.8.5 Damping Elements ................................................................................................... 25

1.8.5.1 Viscous Damping ........................................................................................... 25

1.8.5.2 Coulomb Damping......................................................................................... 27

1.8.5.3 Structural or Hysteretic Damping .................................................................. 29

1.8.5.4 Combination of Damping Elements ............................................................... 30

1.9 Review of Dynamics ....................................................................................................... 32

1.9.1 Newton's Laws of Motion ....................................................................................... 32

1.9.2 Kinematics of Rigid Bodies .................................................................................... 33

1.9.3 Linear Momentum .................................................................................................... 37

1.9.4 Principle of Conservation of Linear Momentum ..................................................... 37

1.9.5 Angular Momentum ............................................................................................... 38

1.9.6 Equations of Motion for a Rigid Body .................................................................... 39

1.9.7 Angular Momentum of a Rigid Body .................................................................... 39

1.9.8 Principle of Work and Energy ............................................................................. 40

1.9.9 Conservation of Energy ......................................................................................... 41

1.9.10 Principle of Impulse and Momentum ................................................................... 42

1.9.11 Mechanical Systems .............................................................................................. 45

1.9.12 Translational Systems .......................................................................................... 46

1.9.13 Rotational Systems .............................................................................................. 47

1.9.14 Translation and Rotational Systems .................................................................... 48

1.9.15 Angular Momentum and Moments of Inertia ...................................................... 48

1.9.16 Geared Systems .................................................................................................... 52

1.10 Lagrange's Equation ..................................................................................................... 59

1.10.1 Degrees of Freedom ............................................................................................... 59

1.10.2 Generalized Coordinates ..................................................................................... 60

1.10.3 Constraints ............................................................................................................. 61

1.10.4 Principle of Virtual Work ...................................................................................... 63
# Road Vehicle Dynamics

## Chapter 1: D'Alembert's Principle
1.10.5 D'Alembert's Principle .............................................. 68
1.10.6 Generalized Force .................................................. 71
1.10.7 Lagrange's Equations of Motion ................................. 74
1.10.8 Holonomic Systems .................................................. 74
1.10.9 Nonholonomic Systems ............................................ 76
1.10.10 Rayleigh's Dissipation Function ................................. 78

## Chapter 2: Analysis of Dynamic Systems
2.1 Introduction ............................................................. 91
2.2 Classification of Vibrations .......................................... 91
2.3 Classification of Deterministic Data ............................... 92
2.3.1 Sinusoidal Periodic Data ......................................... 93
2.3.2 Complex Periodic Data ............................................ 94
2.3.3 Almost Periodic Data ............................................... 95
2.3.4 Transient Nonperiodic Data ...................................... 95
2.4 Linear Dynamic Systems ............................................. 97
2.4.1 Linear Single-Degree-of-Freedom System ...................... 98
2.4.2 Free Vibration of a Single-Degree-of-Freedom System ...... 98
2.4.3 Forced Vibration of a Single-Degree-of-Freedom System ... 102
2.4.4 Linear Multiple-Degrees-of-Freedom System .................. 109
2.4.5 Eigenvalues and Eigenvectors: Undamped System .......... 110
2.4.6 Eigenvalues and Eigenvectors: Damped System ............... 115
2.4.7 Forced Vibration Solution of a Multiple-Degrees-of-Freedom System ......................................................... 119
2.5 Nonlinear Dynamic Systems ........................................... 123
2.5.1 Exact Methods for Nonlinear Systems ......................... 124
2.5.2 Approximate Methods for Nonlinear Systems ............... 128
2.5.2.1 Iterative Method ............................................. 128
2.5.2.2 Ritz Averaging Method ...................................... 131
2.5.2.3 Perturbation Method ......................................... 134
2.5.2.4 Variation of Parameter Method .............................. 140
2.5.3 Graphical Method .................................................. 142
2.5.3.1 Phase Plane Representation .................................. 142
2.5.3.2 Phase Velocity ................................................ 142
2.5.3.3 Pell’s Method .................................................. 144
2.5.4 Multiple-Degrees-of-Freedom Systems ......................... 146
2.6 Random Vibrations .................................................... 147
2.6.1 Probability Density Function .................................... 149
2.6.2 Autocorrelation Function ......................................... 152
2.7 Gaussian Random Process .......................................... 153
2.7.1 Fourier Analysis ................................................... 154
2.7.1.1 Fourier Series .............................................. 154
2.7.1.2 Fourier Integral ............................................. 155
2.7.2 Response of a Single-Degree-of-Freedom Vibrating System ................................................................. 157
2.7.2.1 Impulse Response Method .................................. 158
2.7.2.2 Frequency Response Method ............................... 160
2.7.3 Power Spectral Density Function ................................ 163
2.7.4 Joint Probability Density Function ............................. 165
2.7.5 Cross-Correlation Function ...................................... 166
2.7.6 Application of Power Spectral Densities to Vehicle Dynamics .................................................. 166
2.7.7 Response of a Single-Degree-of-Freedom to Random Inputs ............................................. 168
2.7.8 Response of Multiple-Degrees-of-Freedom Systems to Random Inputs .................................. 170

2.8 Summary ................................................................................................................................. 174
2.9 References ............................................................................................................................ 174

Chapter 3 Tire Dynamics ............................................................................................................. 177
3.1 Introduction ............................................................................................................................. 177
3.2 Vertical Dynamics of Tires ..................................................................................................... 180
3.2.1 Vertical Stiffness and Damping Characteristics of Tires ..................................................... 180
3.2.2 Vertical Vibration Mechanics Models of Tires ...................................................................... 181
3.2.2.1 Point Contact Model of Tires ....................................................................................... 181
3.2.2.2 Fixed Contact Patch Model of Tires ............................................................................. 182
3.2.2.3 Time-Varying Contact Patch Model of Tires ................................................................. 183
3.2.3 Enveloping Characteristics of Tires ...................................................................................... 185
3.3 Tire Longitudinal Dynamics .................................................................................................... 186
3.3.1 Tire Rolling Resistance ........................................................................................................ 187
3.3.2 Rolling Resistance of the Tire with Toe-In .......................................................................... 188
3.3.3 Rolling Resistance of the Turning Wheel ............................................................................ 189
3.3.4 Longitudinal Adhesion Coefficient ...................................................................................... 191
3.3.5 Theoretical Model of Tire Longitudinal Force Under Driving and Braking ......................... 194
3.4 Tire Lateral Dynamics ............................................................................................................. 196
3.4.1 Tire Cornering Characteristics ............................................................................................ 196
3.4.2 Mathematical Model of the Tire Cornering Characteristic .................................................... 198
3.4.2.1 Simplified Mathematical Model of the Tire Cornering Characteristic ......................... 199
3.4.2.2 Cornering Characteristic with Lateral Bending Deformation of the Tire Case ............. 204
3.4.3 Rolling Properties of Tires .................................................................................................. 208
3.4.3.1 Cambered Tire Models .................................................................................................. 209
3.4.3.2 Cambered Tire Model with Roll Elastic Deformation of the Tire Carcass ................. 211
3.5 Tire Mechanics Model Considering Longitudinal Slip and Cornering Characteristics .......... 211
3.5.1 C.G. Gim Theoretical Model ............................................................................................... 212
3.5.2 K.H. Guo Tire Model .......................................................................................................... 214
3.5.2.1 Steady-State Simplified Theoretical Tire Model .............................................................. 214
3.5.2.2 Nonsteady-State Semi-Empirical Tire Mechanics Model ............................................. 219
3.5.3 H.B. Pacejka Magic Formula Model .................................................................................. 224
3.6 References ............................................................................................................................... 228

Chapter 4 Ride Dynamics ............................................................................................................ 231
4.1 Introduction ............................................................................................................................. 231
4.2 Vibration Environment in Road Vehicles .............................................................................. 233
4.2.1 Vibration Sources from the Road ....................................................................................... 233
4.2.1.1 Power Spectral Density in Spatial Frequency ................................................................. 233
4.2.1.2 Power Spectral Density in Temporal Frequency ...........................................237
4.2.2 Vehicle Internal Vibration Sources .................................................................239
  4.2.2.1 Vibration Sources from the Powerplant .......................................................239
    4.2.2.1.1 Coordinates and Powerplant Modes ......................................................239
    4.2.2.1.2 Vibration Sources from Engine Firing Pulsation ......................................242
    4.2.2.1.3 Vibration Sources from Powerplant Inertia Forces and Moments ......................244
    4.2.2.1.4 Powerplant Isolation Design ..............................................................244
  4.2.2.2 Vibration Sources from the Driveline .......................................................252
    4.2.2.2.1 Driveline Imbalance ..............................................................................253
    4.2.2.2.2 Gear Transmission Error .........................................................................255
    4.2.2.2.3 Second Order Excitation .........................................................................256
    4.2.2.2.4 Driveshaft Modes and Driveline Modes ...................................................257
  4.2.2.3 Vibration Sources from the Exhaust System ...............................................257
4.3 Vehicle Ride Models .........................................................................................261
  4.3.1 Quarter Car Model .........................................................................................262
    4.3.1.1 Modeling for the Quarter Car Model ............................................................262
    4.3.1.2 Modal Analysis for the Quarter Car Model ..................................................264
    4.3.1.3 Dynamic Analysis for the Quarter Car Model ..............................................266
      4.3.1.3.1 Transmissibility Between the Body Response and Road Excitation .............266
      4.3.1.3.2 Transmissibility Between the Body Response and Vehicle Excitation ..........271
      4.3.1.3.3 Dynamic Response at Random Input .....................................................272
  4.3.2 Bounce-Pitch Model .......................................................................................274
  4.3.3 Other Modeling ..............................................................................................281
4.4 Seat Evaluation and Modeling ..........................................................................282
  4.4.1 Introduction ......................................................................................................282
  4.4.2 SEAT Value .....................................................................................................283
  4.4.3 Seat Velocity ...................................................................................................285
  4.4.4 Linear Seat Modeling and Transmissibility .......................................................286
  4.4.5 Nonlinear Seat Modeling and Transmissibility ................................................286
4.5 Discomfort Evaluation and Human Body Model .................................................290
  4.5.1 Discomfort and Subjective Evaluation ..............................................................290
  4.5.2 Objective Evaluation of Ride Discomfort ........................................................292
    4.5.2.1 Weighted Root-Mean-Square Method .........................................................292
    4.5.2.2 Objective Evaluation by the Vibration Dose Value .......................................293
  4.5.3 Linear Human Body Modeling .........................................................................294
  4.5.4 Objective Evaluation by Nonlinear Seat–Human Body Modeling .....................295
4.6 Active and Semi-Active Control .......................................................................298
  4.6.1 Introduction ......................................................................................................298
  4.6.2 Basic Control Concepts ...................................................................................298
  4.6.3 Active Control ..................................................................................................299
  4.6.4 Semi-Active Control ........................................................................................302
4.7 Summary .............................................................................................................307
4.8 References ..........................................................................................................307
Chapter 5 Vehicle Rollover Analysis

5.1 Introduction ................................................................................. 311
  5.1.1 Rollover Scenario ............................................................... 311
  5.1.2 Importance of Rollover ......................................................... 314
  5.1.3 Research on Rollover ........................................................... 314
  5.1.4 Scope of This Chapter .......................................................... 315

5.2 Rigid Vehicle Rollover Model .................................................. 316
  5.2.1 Rigid Vehicle Model ............................................................ 316
  5.2.2 Steady-State Rollover on a Flat Road ................................. 317
  5.2.3 Tilt Table Ratio ................................................................. 318
  5.2.4 Side Pull Ratio ................................................................. 320

5.3 Suspended Vehicle Rollover Model ....................................... 321
  5.3.1 Steady-State Rollover Model for a Suspended Vehicle .......... 321
  5.3.2 Contribution from the Tire Deflection ............................... 323
  5.3.3 Contribution from the Suspension Deflection .................... 324
  5.3.4 Parameters Influencing the Suspended Rollover Model ........ 326

5.4 Dynamic Rollover Model ......................................................... 333
  5.4.1 Rigid Dynamic Model ......................................................... 333
  5.4.2 Dynamic Rollover Model for a Dependent Suspension Vehicle .................................................................................. 334
  5.4.3 Dynamic Rollover Model for an Independent Suspension Vehicle .................................................................................. 336
  5.4.4 Rollover Simulation Tools .................................................. 336

5.5 Dynamic Rollover Threshold .................................................. 338
  5.5.1 Dynamic Stability Index ...................................................... 338
  5.5.2 Rollover Prevention Energy Reserve ................................. 339
  5.5.3 Rollover Prevention Metric ............................................... 340
  5.5.4 Critical Sliding Velocity ................................................... 340

5.6 Occupant in Rollover ............................................................... 341
  5.6.1 Overview of the Occupant and Rollover ......................... 341
  5.6.2 Testing of an Occupant Model ......................................... 342
  5.6.3 Simulation of Occupant Rollover ..................................... 343

5.7 Safety and Rollover Control .................................................... 345
  5.7.1 Overview of Rollover Safety ........................................... 345
  5.7.2 Sensing of Rollover ......................................................... 349
  5.7.3 Rollover Safety Control ................................................... 350

5.8 Summary ................................................................................ 352

5.9 References ............................................................................ 353

Chapter 6 Handling Dynamics ..................................................... 357

6.1 Introduction ............................................................................. 357
  6.1.1 Tire Cornering Forces ....................................................... 358
  6.1.2 Forces and Torques in the Tire Contact Area .................... 360

6.2 The Simplest Handling Models—Two-Degrees-of-Freedom
  Yaw Plane Model ..................................................................... 361

6.3 Steady-State Handling Characteristics ................................ 365
  6.3.1 Yaw Velocity Gain and Understeer Gradient .................... 365
    6.3.1.1 Neutral Steer ......................................................... 367
    6.3.1.2 Understeer ........................................................ 367
    6.3.1.3 Oversteer .......................................................... 368
  6.3.2 Difference Between Slip Angles of the Front and
    Rear Wheels ....................................................................... 368
9.1.3.4 Braking ........................................... 535
9.1.3.5 Performance ..................................... 536

9.2 Steering and Braking ................................ 536
9.2.1 Simple Braking and Steering on a Horizontal Road 537
9.2.2 Optimal Braking Performance Under Steering .... 542
9.2.2.1 Front Lock-Up .................................. 544
9.2.2.2 Rear Lock-Up .................................. 545

9.3 Steering and Acceleration .......................... 548
9.3.1 Simple Acceleration and Steering on a Horizontal Road 548
9.3.2 Optimal Acceleration Performance Under Steering .... 550
9.3.2.1 Front Skid ....................................... 551
9.3.2.2 Rear Skid ....................................... 552

9.4 Vehicle Critical Speed ............................... 556
9.5 Vehicle Stability ..................................... 558
9.6 Summary ................................................ 560
9.7 References ............................................. 560

Chapter 10 Accident Reconstruction .......................... 563
10.1 Introduction and Objectives ........................ 563
10.2 Basic Equations of Motion ............................ 564
10.3 Drag Factor and Coefficient of Friction ............ 568
10.5 Driver Perception and Response ..................... 573
10.6 Engineering Models and Animations ................ 575
10.6.1 Function of Accident Scene Models ............. 575
10.6.2 Model Application ................................ 576
10.6.3 Reconstruction Animations ..................... 576

10.7 Lane Change Maneuver Model ....................... 577
10.8 Speed Estimates for Fall, Flip, or Vault .......... 584
10.8.1 Fall ................................................ 584
10.8.2 Flip ............................................... 587
10.8.3 Vault ............................................. 587

10.9 Speed Estimates from Yaw Marks .................... 588
10.10 Impact Analysis ...................................... 591
10.10.1 Straight Central Impact .......................... 591
10.10.2 Noncentral Collisions ............................ 593
10.10.3 Crush Energy and ΔV ............................. 594

10.11 Vehicle–Pedestrian Collisions ....................... 598
10.11.1 Pedestrian Trajectories ......................... 599
10.11.2 Mathematical and Hybrid Models ............... 601

10.12 Accident Reconstruction Software ................ 608
10.12.1 Software Acronyms: REC-TEC with DRIVE³ and MSMAC<sup>RT</sup> ................. 608
10.12.2 VCRware ........................................... 609
10.12.3 CRASHEX ......................................... 611
10.12.4 ARSoftware ........................................ 612
10.12.5 Engineering Dynamics Corporation ............. 613
10.12.6 Macinnis Engineering Associates (MEA) and MEA Forensic Engineers & Scientists ....... 615
10.12.7 Maine Computer Group .......................... 616
10.12.8 McHenry Software, Inc .......................... 617
10.12.9 Software Acronym: VDANL ....................... 620
10.12.10 Expert AutoStats®—Vehicle Dimension-Weight-Performance Data .......................................................... 622
10.12.11 Other Accident Reconstruction Software Sites .................. 622
10.13 Low-Speed Sideswipe Collisions ..................................... 623
  10.13.1 Funk-Cormier-Bain Model ........................................ 624
  10.13.2 Modeling Procedure ............................................... 625
10.14 Summary ................................................................... 629
10.15 References .................................................................. 629

Appendix A Vector Algebra .................................................... 635
  A.1 Real and Complex Vectors .............................................. 635
  A.2 Laws of Vector Operation ............................................. 636
  A.3 Linear Dependence ...................................................... 636
  A.4 Three-Dimensional Vectors .......................................... 637
  A.5 Properties of the Scalar Product of Vectors ..................... 638
  A.6 Direction Angles ........................................................ 638
  A.7 Vector Product .......................................................... 638
  A.8 Derivative of a Vector ................................................... 639
  A.9 References ............................................................... 640

Appendix B Matrix Analysis .................................................. 643
  B.1 Introduction ................................................................ 643
  B.2 Definitions of Matrices ................................................ 643
  B.3 Matrix Operations ....................................................... 648
  B.4 Matrix Inversion ........................................................ 651
  B.5 Determinants ............................................................ 652
  B.6 More on Matrix Inversion ............................................. 657
  B.7 System of Algebraic Equations ...................................... 660
  B.8 Eigenvalues and Eigenvectors ...................................... 664
  B.9 Quadratic Forms ....................................................... 668
  B.10 Positive Definite Matrix .............................................. 668
  B.11 Negative Definite Matrix ............................................. 670
  B.12 Indefinite Matrix ....................................................... 671
  B.13 Norm of a Vector ....................................................... 671
  B.14 Partitioning of Matrices .............................................. 672
  B.15 Augmented Matrix ..................................................... 673
  B.16 Matrix Calculus ......................................................... 674
  B.17 Summary .................................................................. 675
  B.18 References ............................................................... 675
  B.19 Glossary of Terms ..................................................... 676

Appendix C Laplace Transforms ............................................. 679
  C.1 Laplace Transformation ................................................ 679
  C.2 Existence of Laplace Transform .................................... 680
  C.3 Inverse Laplace Transform .......................................... 681
  C.4 Properties of the Laplace Transform .............................. 681
    C.4.1 Multiplication by a Constant ................................... 681
    C.4.2 Sum and Difference .............................................. 682
  C.5 Special Functions ...................................................... 682
    C.5.1 Exponential Function ............................................. 682
    C.5.2 Step Function ..................................................... 682
Foreword

My career in vehicle dynamics began more than thirty years ago in what I now see with the benefit of hindsight to have been the Dark Ages. In those days, we were guided as much by intuition and experience as by science. Our ability to deductively model the physical laws of dynamics was rather limited, and our ability to measure the dynamics of a vehicle was somewhat primitive, as was our ability to predict the performance of a particular geometry and set of components. As a consequence, we had to rely heavily on tuning experts to iron out the problems that we had inadvertently designed into the vehicle—a luxury that as an industry we can no longer afford.

In the intervening years, the science of vehicle dynamics has undergone a revolution. A great deal of excellent work has been done in the field of geometry, and the availability of virtually high-powered numerically intensive computing capability has enabled us to vastly expand our understanding of vehicle dynamics through the construction of theoretical models. These analytical methods are a mix of deductive, empirical, and hybrid types that now can help us come much closer to the optimal design prior to the prototype stage. This includes not only the design concept but also the parameterization of the design to reduce error states. The result is that the design and development processes have become much more streamlined and efficient, and the end product has improved greatly. Today’s cars undoubtedly are much safer and more enjoyable to drive than those of thirty years ago, and much of this improvement can be traced back to the more extensive understanding and use of analytical methods.

But we cannot be complacent. We still must do time-consuming tuning, and some failure modes are still detected after the point of initial release. Our quest to build safer cars in a world of increasing competition and cost constraints will be possible only with the application of analytical predictive methods. New product cycle times are shrinking, model ranges are expanding, and the cost of building prototypes is rising, making it imperative that we find faster and more efficient ways of working.

Despite all the technological advances we have made, vehicle dynamics remains an art as much as a science. It requires the exercise of good judgment in matters such as how to translate a customer’s requirements into objective metrics that engineers can measure, and how to balance many competing requirements—such as ride, handling, and braking—within a package that is both affordable and consistent with the distinctive attributes of a particular brand.

This book will not teach you how to exercise good judgment—only first-hand experience can do that—but it will give you an excellent foundation in this fascinating and important field. A combination of theoretical fact and practical insight, this book will help you to play your part in meeting the challenges of vehicle dynamics in the twenty-first century.

Professor Richard Parry-Jones, CBE
Group Vice President, Global Product Development
Chief Technical Officer
Ford Motor Company
Preface

This book provides a fundamental understanding of how physical laws, human factor considerations, and design choices interact to determine the ride, handling, braking, and accelerating behavior of road vehicles used for personal transportation. It presents the dynamics of road vehicle systems from a perspective that unifies the treatment of the causes of physical events with a treatment of the reasons for physical functions. Emphasis is placed on the observation that driver vehicle system behavior is constrained by physical law, but the driver’s objectives and the designer’s goals ultimately will determine how the system is utilized and configured.

Currently, the driver’s skill, situation awareness, and knowledge largely determine control qualities in service. However, systems aimed at aiding the driver in controlling the vehicle are being developed and sold, such as anti-lock braking systems, adaptive cruise control systems, and anti-spinout directional control systems. These new systems are based on control system concepts, and they contain sensors, units for processing sensor information, and actuators for implementing desired control actions. These control systems perform functions that are difficult or tedious for the driver to perform. In the context of this book, the point is that the reasons for these functions can be stated in analytical terms. Related to this point, a goal of this book is to provide a conceptual approach that can aid professionals and those in the academic world in using ideas similar to those used in control system design to plan and envision functional enhancements to the vehicle system. A key element of this approach lies in translating the reason for a new functional capability into analytical rules that can be employed in a control system that makes the vehicle simultaneously safer, more comfortable in which to ride, and/or easier to drive.

Practical approaches involving pragmatic considerations of reason and cause are treated for each of the traditional vehicle dynamics areas—ride, handling (steering), braking, and accelerating behavior. In each of these areas, this book covers the analysis of vehicle dynamics issues using physical principles, and it considers vehicle design in the sense that a design is a plan based on an understanding of the physical properties of the vehicle, as well as an understanding of what the vehicle is expected to do.

In the chapters related to handling, braking, and accelerating, the individual control inputs are viewed as a means of communication between the driver, or an automatic control system, and the basic vehicle. From a systems perspective, the driver or control system activates the controls to obtain vehicle system dynamics that fit the driver’s or the control system’s plans and expectations. Accordingly, this book not only focuses inwardly on mechanisms within the vehicle but also looks outwardly at broader considerations involving what the driver desires and how to achieve it.

Even in areas such as ride and roll where the driver has no direct means of influencing the motions caused by hitting holes and bumps, emphasis is placed on driver concerns with control and safety. A common theme is to indicate how driver and passenger considerations influence the design characteristics of motor vehicle systems.

This book is written for vehicle designers, developers, evaluators, and both senior undergraduate students and graduate students. Previous knowledge of differential
There are probably a handful of books on vehicle dynamics. Some of these books are written at a fundamental level that may not meet ambitious engineering program requirements. Others are specialized in certain fields of vehicle dynamics, including modeling and simulation. In this book, we attempt to strike a balance between theory and practice, fundamentals and advanced subjects, and generality and specialization. However, the book focuses on road vehicles, and little treatment is made for off-road vehicles. Road vehicles that frequently are driven off-road (e.g., sport utility vehicles [SUVs]) are covered in this book. The book also emphasizes the use of cars and light trucks, with little treatment of vehicles with trailers. We had this emphasis in mind mainly for two reasons:

1. Engineers interested in off-road vehicle dynamics or the dynamics of multi-axle vehicles can build on the fundamentals given in this book.

2. Most vehicles produced for the public are classified as cars or light trucks (13 to 16 million such vehicles are sold annually). These also have the most stringent requirements for ride, safety, fuel economy, and other attributes.

We pay particular attention to the issue of safety in this book because it is among the leading (if not the leading) attributes in customers’ minds when purchasing a vehicle. In addition, safety is the attribute loaded with governmental standards that must be met before the vehicle can be offered to the public. One complete chapter is written on accident reconstruction to address legal issues that result from an automotive accident.

This book contains ten chapters with an extended list of appendices. The appendices provide a review of the material needed before studying the book, or they provide other material that ensures consistency of the terms used in this book. Readers will find appendices covering matrix and vector algebra, Fourier series, Laplace transformation, vehicle dynamics terminology, direct numerical integration methods, and conversion of units.

Chapter 1 is an introduction and general review of dynamics. It covers vehicle system classification, dynamic systems and their models, generalized coordinates and degrees of freedom, discrete and continuous systems, vibration analysis, Newton’s laws of motion, kinematics of rigid bodies, concepts of work and energy, impulse, D’Alembert’s principle, Lagrange’s equations of motion, holonomic and nonholonomic systems, and other subjects.

Chapter 2 focuses on the analysis of dynamic systems. Subjects treated include the classification of dynamic system models, constraints, generalized coordinates, degrees of freedom (DOF), classification of deterministic data, linear dynamic systems, free and forced vibrations, eigenvalues and eigenvectors, damping, nonlinear dynamic systems, random vibrations, and methods of analysis.

The forces acting on the vehicle are treated in Chapter 3, which covers tires and aerodynamic forces and moments. Particular emphasis is given to explaining how the tire produces the forces needed to move the vehicle laterally and longitudinally as well as to support the loads. The dynamic performance of the entire vehicle system is limited by what the tires can do. Every aspect of vehicle performance is constrained by what is happening at the tire/road interface. Hence, it is desirable that each subsystem of
the vehicle aids in presenting the tire to the road as favorably as possible. No other component of the vehicle is singled out for separate treatment as that given to the tire because no other component is as important to so many different aspects of riding and driving performance. This book presents empirical and simplified theoretical approaches for representing the input-output characteristics of tires as needed for vehicle dynamic analyses.

Chapter 4 treats the subject of ride dynamics. Issues discussed include sources of vibrations, power spectral density, vehicle ride models, seat evaluation and modeling, discomfort evaluation, human body model, and material on active and semi-active control.

Chapter 5 covers roll dynamics and treats subjects on rollover, including rollover scenarios, rollover modeling, and rollover testing. Rollover modeling includes a suspended vehicle rollover model, steady-state rollover models, and a dynamic rollover model. Rollover testing includes the tilt table ratio (TTR) test and the side pull ratio (SPR) test. This chapter also includes studies on the contribution of tire deflection and suspension deflection to rollover. In addition, rollover control is discussed in this chapter.

Chapter 6 treats handling dynamics. Subjects include steady-state handling, understeer and oversteer, steering sensitivity, roll gain and roll damping, transient response, lateral acceleration and yaw rate response time, and others. Various handling models are presented, including a two-degrees-of-freedom yaw plane and nonlinear models. Safety considerations and overturning limits also are studied.

Chapter 7 discusses braking, spanning both drum and disk brakes. Torque distribution along the brake pads is discussed thoroughly, including the treatment of temperature rise.

Load transfer during braking under the effects of aerodynamic rolling and other forces, as well as the effect of grade, are studied. Optimal braking performance for single- and multi-axle braking is investigated, paying attention to safety. Front and rear lock-ups and anti-lock braking systems (ABS) also are discussed.

In Chapter 8, vehicle acceleration is studied, including load transfer during acceleration under the effect of various forces. Treatment of power and traction limited acceleration also is given. Studies are made on acceleration for various drivetrains, including front-wheel drive, rear-wheel drive, and four-wheel drive. Torque distribution for different drive axles and optimal tractive effort also are treated, as well as discussions on engine power and power transmission with manual, automatic, and other types of transmissions. Finally, safety features are presented, including limited slip axle and traction control.

Chapter 9 introduces total vehicle dynamics, covering both subjective and objective vehicle evaluations, dynamics of combined steering and braking, and combined steering and acceleration.

Chapter 10 presents the use of a wide variety of methodologies and software for accident reconstruction, based on the variation of physical evidence and accident investigative information.

Appendices A through G present a basic review of vector algebra, matrix analysis, Laplace transforms, a glossary of terms, direct numerical integration methods, units and conversions, and accident reconstruction formulae, respectively. Likewise, an extensive bibliography to guide the reader to further sources of information on road vehicle dynamics is provided at the end of the book. Both the S.I. and the U.S./English systems of units have been used throughout this book.
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