Principles of Vibration Analysis

with Applications in Automotive Engineering
Other Related Publications Available from SAE International:

**Vehicle Refinement**  
By Matthew Harrison  
(Product Code: R-364)

**Vehicle Vibration and Sound**  
By Gang Sheng  
(Product Code: R-400)

**Car Suspension and Handling, Fourth Edition**  
By Geoffrey Howard  
(Product Code R-318)

For more information or to order a book, contact SAE International at 400 Commonwealth Drive, Warrendale, PA 15096-0001 USA; phone 877-606-7323 (U.S. and Canada only) or 724-776-4970 (outside U.S. and Canada); fax (724) 776-0790; e-mail CustomerService@sae.org; website http://books.sae.org.
Principles of Vibration Analysis
with Applications in Automotive Engineering

C. Q. Liu and Ronald L. Huston

SAE International
Warrendale, Pennsylvania
USA
Contents

Preface xiii

Chapter One Introduction 1

1.1 What Is Vibration? 1

1.2 Classification of Vibration Studies 2

(1) Vibration Analysis 2

(2) System Design 2

(3) Input Evaluation 2

(4) System Identification 2

1.3 Classification of Vibration 3

(1) Classification Based on Input 3

(2) Classification Based on Output 4

(3) Classification Based on the Degrees of Freedom of the System 7

(4) Classification Based on Differential Equation of Motion of the System 7

1.4 Harmonic Vibration 8

1.5 Harmonic Analysis 10

Problems 16

References 18

Chapter Two Vibration of a Single-Degree-of-Freedom System 21

2.1 Introduction: Modeling 21

2.2 Free Vibration of a Single-Degree-of-Freedom System 24

(1) Governing Differential Equation of Motion 24

(2) Equivalent Spring Stiffness 28

2.3 The Energy Method 33

(1) The Law of Conservation of Energy 33

(2) The Rayleigh Method 34

2.4 Effective Mass 40

2.5 Damped Free Vibration of a Single-Degree-of-Freedom System 41

(1) Damping Forces 41

(2) Differential Equation of Motion 41

(3) Discussion of the Solution 43

Problems 50

References 53
Chapter Three  Harmonically Excited Motion

3.1 Introduction 55
3.2 Forced Harmonic Vibration of Damped Systems 55
3.3 Forced Harmonic Vibration of Undamped Systems 63
   (1) Beating 64
   (2) Resonance 66
3.4 Forced Vibration Caused by Rotating Unbalance 67
3.5 Forced Vibration Caused by Support Motion 69
3.6 Vibration Isolation 72
   (1) Isolation of the Machine from the Foundation 73
   (2) Isolation of the Foundation from the Machine 73
3.7 Damping 76
   (1) Energy Dissipation Due to a Viscous Damping Force 76
   (2) Structural (Hysteretic) Damping 78
   (3) Complex Stiffness and Lost Factor 79
   (4) Sharpness of Resonance 80
3.8 Forced Vibration Under Periodic Excitation 81
3.9 Response to Arbitrary Excitation 82
Problems 87
References 92

Chapter Four  Balancing of Rotors

4.1 Unbalance 93
   (1) Static Unbalance 93
   (2) Dynamic Unbalance 94
4.2 Whirling of Rotating Shaft 95
4.3 Experimental Balancing 98
   (1) The Influence Coefficient Method: Single-Plane Balancing 98
   (2) The Influence Coefficient Method: Two-Plane Balancing 100
4.4 Inertia Forces in a Single-Cylinder Engine 104
4.5 Inertia Forces and Couples of Multicylinder Engines 110
4.6 Balancing of Crankshafts 120
Problems 126
References 133
Chapter Five  Multi-Degree-of-Freedom Systems  
5.1  Introduction  
5.2  Two-Degree-of-Freedom Systems  
   (1)  Equations of Motion  
   (2)  Characteristic Equation and Natural Frequency  
   (3)  Mode Shapes  
5.3  Eigenvalues and Eigenvectors  
5.4  Orthogonal Properties of the Modal Vectors  
5.5  Zero Eigenvalues and Repeated Roots  
   (1)  Zero Eigenvalues  
   (2)  Repeated Roots  
5.6  Response of a System to Initial Conditions  
5.7  Coordinate Coupling  
5.8  Undamped Systems and Coordinate Decoupling  
   (1)  Forced Response and Coordinate Decoupling  
   (2)  Initial Value Problem and Coordinate Decoupling  
5.9  Damped Systems and Coordinate Decoupling  
   (1)  Coordinate Decoupling with Forced Response  
   (2)  Coordinate Decoupling with Initial Value Problems  
5.10  Lagrange's Equations  
   (1)  Generalized Coordinates and Virtual Displacements  
   (2)  Virtual Work and Generalized Forces  
   (3)  Lagrange's Equations  
5.11  Kane's Equations  
   (1)  Partial Velocities and Partial Angular Velocities  
   (2)  Inertia Forces  
   (3)  Generalized Applied Forces  
   (4)  Kane's Equations  
Problems  
References  

Chapter Six  Numerical Methods  
6.1  Introduction  
6.2  Various Eigenvalue Analyses  
   (1)  The Generalized Eigenvalue Problem  
   (2)  The Algebraic Eigenvalue Problem  
Problems  
References
(3) The Symmetric Eigenvalue Problem 180
(4) The Standard Eigenvalue Problem in State Space 183
(5) The Generalized Eigenvalue Problem in State Space 185
6.3 Numerical Evaluation of the Time Response 197
Problems 210
References 214

Chapter Seven  Transfer Functions and Frequency Response Functions 217
7.1 Introduction 217
7.2 The Laplace Transformation and Fourier Transformation 217
(1) The Laplace Transform 217
(2) The Fourier Transform 219
7.3 Laplace Domain: Transfer Function 220
7.4 Frequency Domain: Frequency Response Function 223
(1) Receptance (or Admittance, or Dynamic Compliance) 223
(2) Mobility 223
(3) Accelerance (or Inertance) 223
(4) The Bode Diagrams 226
(5) The Nyquist Diagrams 228
7.5 Time Domain: Impulse Response Function 230
7.6 FRFs of Multi-Degree-of-Freedom Systems 234
(1) Laplace Domain: Transfer Function Matrix 235
(2) Frequency Domain: Frequency Response Function Matrix 238
(3) Time Domain: Impulse Response Functions 241
7.7 Determination of Residues and Poles in State Space 248
Problems 254
References 257

Chapter Eight  Lumped Parameter Systems 259
8.1 Introduction 259
8.2 The Flexibility and Stiffness Matrices 259
8.3 Maxwell’s Reciprocity Theorem 261
8.4 Eigenvalue Analysis 265
8.5 Dunkerley’s Equation 269
8.6 Rayleigh Principle 271
8.7 Method of Matrix Iteration 274
8.8 Determining of Higher-Order Modes and Frequencies 277
8.9 Transfer Matrix Method  279
(1) A Spring-Mass System  280
(2) Torsional Systems  286
(3) A Geared System  289
(4) Branched Systems  290
8.10 Beams Modeled as Lumped Masses  292
(1) Flexural Vibration  292
(2) Rotating Beams  301
Problems  304
References  308

Chapter Nine  Continuous Systems  309
9.1 Introduction  309
9.2 Lateral Vibration of a String  310
(1) d'Alembert's Solution  310
(2) Separation of Variables  312
9.3 Longitudinal Vibration of Rods  315
9.4 Torsional Vibration of Rods  320
9.5 Lateral Vibration of Beams  323
9.6 Orthogonal Relations  331
9.7 Effects of Rotary Inertia, Shear, and Axial Loading  336
(1) The Effect of Rotary Inertia and Shearing Deformation  336
(2) The Effect of Axial Force on Lateral Vibration  339
9.8 Forced Vibration  340
9.9 Ritz Method  344
Problems  348
References  351

Chapter Ten  Engine Mounting Systems  353
10.1 Introduction  353
10.2 Inertia Properties of an Engine  354
(1) Reference Frame and Direction Cosines  358
(2) Inertia Matrix  358
(3) Mass Center  361
10.3 Orientation Angles and Transformation Matrix  363
10.4 Equations of Motion  372
(1) Forces and Torques Produced by an Engine Mount
due to Translational Motion of the Engine  374
Chapter Eleven  Experimental Modal Analysis

11.1 Introduction  405
11.2 Modal Analysis Theory  406
11.3 The Complex Exponential Algorithm  408
11.4 The Least-Squares Complex Exponential Method  412
11.5 The Polyreference Time Domain Method  412
11.6 The Ibrahim Time Domain Method  419
11.7 The Eigensystem Realization Algorithm  422
11.8 Estimation of Modal Vectors  426
11.9 Estimation of Frequency Response Functions  428
   (1) $\hat{H}_1$ Estimator of FRF  430
   (2) $\hat{H}_2$ Estimator of FRF  431
   (3) $\hat{H}_v$ Estimator of FRF  432
11.10 Coherence Function  433
   (1) Ordinary Coherence Function  433
   (2) Multiple Coherence Function  434
   (3) Partial Coherence Function  435
11.11 Discrete Fourier Transfer (DFT) and Fast Fourier Transfer (FFT)  436
   (1) Aliasing  438
   (2) Leakage  438
11.12 Window Functions  440
11.13 Averaging  442
   (1) Linear Average  443
   (2) Stable Average  443
11.14 Overlapping Signal Analysis 443
11.15 Modal Data Acquisition 444
   (1) Test Structure 445
   (2) Exciter 447
   (3) Data Acquisition System 451
   (4) Analyzer 453
11.16 Operational Modal Analysis 453
   (1) Stochastic Subspace Identification Method 454
   (2) The Natural Excitation Technique (NExT) 457
11.17 Running Modes Analysis 458
Problems 460
References 463

**Chapter Twelve** Special Topics 467
12.1 Introduction 467
12.2 Complex Structure Analysis Using the FRFs of Substructures 467
   (1) Equations of Motion 468
   (2) Solution Method 469
12.3 Translational Vibration Absorber 475
   (1) Absorber Tuning Based on the Primary System FRF 476
   (2) Special Cases: A Single-Degree-of-Freedom System 480
12.4 Torsional Vibration Absorber 486
12.5 Experimental Measurement of the Torsional FRF 487
12.6 HYFEX Method 488
12.7 Sensitivity Analysis 495
   (1) Eigenvalue Derivatives 496
   (2) Eigenvector Derivatives 497
References 503

**Appendix A** 507

**Appendix B** 509

**Appendix C** 513

**Notation** 519

**Index** 537
Preface

Over the years there have been many outstanding books written on vibrations, including textbooks, books on analysis and design, and theoretical treatises. Why then should there be another, and, specifically, why this book?

There are several reasons: First, vibrations, as with all science and engineering subjects, is continually evolving. As new discoveries and new analyses and new experimental procedures are developed, there is an ongoing need to document these advances.

Next, with the evolution in the subject, many of the older procedures become obsolete. Traditional topical emphases become less important, and are superseded by advances in computational and measurement methods. Consequently, new topical emphases are emerging. There is a need to document these new methods for students and for practicing engineers.

Finally, it is always helpful to have a state-of-the-art treatise on fundamentals enabling future research and application.

We wrote this book attempting to satisfy each of these needs. The book summarizes basic concepts and established analytical methodologies. We then illustrate these ideas and procedures via worked-out examples. Numerous examples using Matlab are presented throughout the text. The various chapters contain problems (with answers) for readers studying the subject for the first time, and for those seeking additional expertise and/or review. In the later chapters, we illustrate application in automotive engineering with a focus upon engine balancing and engine mounting systems. Finally, the book is intended to be a quick reference for current (circa 2010) modeling and analysis methods.

The book is divided into twelve chapters, with the first of these providing an introduction to vibration terminology. In the second chapter we review the fundamentals of single-degree-of-freedom systems, and in the third chapter we consider harmonically excited motion.

The fourth chapter applies these concepts with rotors, outlining balancing methods.

In the fifth chapter we study multi-degree-of-freedom systems, and then we devote a chapter to numerical methods. In the seventh chapter we introduce the concept of transfer functions and frequency response functions together with a detailed discussion about these functions.

Chapter eight explores lumped parameter modeling and chapter nine follows with analyses of continuous systems.

Chapter ten provides an extensive exposition of engine mounting systems.
Chapter eleven summarizes experimental modal analysis methods, including the basic theory and an account of signal processing and test procedures.

In the final chapter, we conclude with some special topics, such as the frequency response function (FRF)-based substructure method, the hybrid finite element and experimental (HYFEX) method, system sensitivity analysis, and vibration absorber tuning.

Prior to writing this book we were fortunate to have studied under and been associated with academicians and engineers in industry who have significantly advanced the state of the art through their research, development, application, and exposition of both theoretical and experimental breakthroughs. We are particularly appreciative of the inspiration provided by the innovations of faculty (David L. Brown and Randall J. Allemang) and students at the Structural Dynamics Research Laboratory (SDRL) at the University of Cincinnati, and of engineers in local industries, and particularly in the automotive industry. We have relied heavily upon these sources for our exposition.

We have also, of course, relied upon many others, as cited in the references and permissions. We have attempted to acknowledge all sources, but undoubtedly have missed many which should have been cited. We apologize for any such omissions.

A number of the examples in this book have originated with the references listed in the chapters. Although we attempt to acknowledge these sources, the examples are not direct quotations. Instead they are usually done differently and often more extensively than in the quoted sources.

We appreciate the assistance of Charlotte Better in preparing the manuscript. We also appreciate the assistance of Aaron Lock in preparing the artwork for this book.

Finally, we are grateful to the Society of Automotive Engineers (SAE) for their interest and their acceptance of our work.

C. Q. Liu
Ronald L. Huston