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Vehicle Accident Analysis and Reconstruction Methods

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A great deal of support was provided by our families during the preparation of this work. Our wives, Carol Brach and Paula Brach, encouraged this endeavor and shared in the effort required to complete the task. Matt’s children, Elizabeth, Olivia, and Daniel, sacrificed family time to permit writing to be done. This book is dedicated to them.
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  Notation
Vehicle accident reconstruction is a discipline which has developed and grown enormously over the (nearly) forty years that it has been my good fortune to be involved in it. In the early 1970s the methods were crude, being largely confined to simple calculations on skidding and stopping distances and on how far this vehicle or that person would move in a given time. It was as well that the calculations were simple, since slide rules were the only practicable portable calculator, and personal computers were an improbable dream. In those days the ordinary practitioner knew very little about such things as tire dynamics, road surface properties, vehicle crush behavior, or what happens when a pedestrian is hit by a car. But gradually accident reconstructionists picked up knowledge on these matters from various fields of learning—vehicle and highway engineering, safety research, driver psychology, trauma medicine—and at the same time the means of handling it, in the shape of calculators, computers, and eventually the internet came into being. A good example is the CRASH program, developed for NHTSA as a road safety research tool. Although by around 1980 it was being recognised as something that reconstructionists could use, it required a mainframe computer, and even then there were no graphics, so the scope for misusing it was considerable. Yet by 1990 a number of versions of it, complete with graphics and much greater flexibility than the original, were available for use in portable personal computers.

But a problem throughout this period was how to get a deep but practical knowledge of what was an ever-widening field of study. The admirable manuals published by Northwestern University both then and now covered a great deal of the field at a practical but relatively shallow level, being primarily directed at police officers rather than engineers. Anyone wishing to delve deeper was confronted by daunting volumes on the rigorous analysis of tire and vehicle dynamics, or the construction of highways, or
the medical details of occupant and pedestrian trauma. One response was the advent of accident reconstruction societies and conferences for the dissemination of information and pieces of research: as far as I can ascertain, the first society with a wide reach was IAARS, founded in 1980, while notable among conferences has been the annual Accident Reconstruction Session at the SAE World Congress, which began in 1987. But despite all this, there was still the lack of a scholarly and mathematically rigorous text for the reconstructionists. This is where Ray and Matt Brach come in.

I first met Ray Brach in the late 1980s, when he visited me in London. I immediately recognised him as a practical as well as academic engineer, with a deep interest in solving everyday problems, notably vehicle collisions. I am pleased to say that the acquaintance has grown over the years, with getting to know his son Matt as well, and more recently through their management of the SAE Congress sessions. A book by them was therefore something to look forward to, and the first edition, published in 2005, did not disappoint. But a considerable bonus with the book was a training course, and I had the pleasure of hosting Ray and Matt when they presented it to a group of investigators (myself included) from TRL in 2006.

A new edition is, of course, very welcome. The investigations which the authors have carried out into tire models, as used in the various simulation programs and now brought into Chapter 2, are particularly interesting to anyone hoping to simulate more extreme vehicle motions, while the new chapter on articulated vehicle collisions gives us a handle on a (for some of us) near intractable problem. Collisions at railroad grade crossings (the new Chapter 12) are less of an issue in this writer's country, but then the treatment is equally valid for any vehicle-vehicle conflict at an intersection: and the unexpected and fascinating exposition on train horns is a fine example of how this field just gets wider and wider.

This new edition is a most welcome addition to the accident reconstructionist’s library and will be a valuable source of guidance and information.

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December 2010
This book remains unique as a presentation of methods of accident reconstruction — scientific, engineering, and mathematical methods. In contrast to many other books on this subject, the methods presented here are supported by references and/or data to establish their validity. It is not simply a second edition of an existing book, but is a revised and enhanced version. Typographical errors in the first edition have been corrected. Additional examples have been included in many of the chapters. The Glossary has been updated, particularly to include many new acronyms that have emerged with the use of event data recorders (EDRs). More noteworthy, however, is the improved coverage of tire forces; additional experimental data are presented and the topic of the tire ellipse/circle is taken out of the idealized realm, explained, and placed into a practical, realistic format based on experimental data.

Tables and a nomograph for the use of frictional drag coefficients for sliding tires have been added. With appropriate modification, these can be used with applications involving antilock braking systems. Frictional drag coefficients for heavy trucks differ from those of light vehicles; this topic is now covered, with data and references included.

Two new chapters have been added. Articulated vehicles are ever-present on our roads and are overly involved in serious crashes. The analysis of collisions of articulated vehicles requires special concepts and methods. The equations of the mechanics of such collisions are covered in a new chapter of this edition. The topic of the conflict of vehicles (road vehicles or rail trains) approaching each other at an intersection of roads or at a railroad grade crossing has received little or no coverage in other books. A new chapter in this edition presents methods for analyzing and reconstructing such events. In addition, this chapter covers the acoustics of train horns to allow estimation of sound levels from locomotives at grade crossings.

Overall, the authors hope that this book will assist users of the methods presented here by placing their reconstructions of vehicle accidents on a firmer, more scientific foundation. As before, feedback is welcome, particularly if any errors are found.
More than 19,000 registered junkyards (junkyards.com) operate in the United States. Each can have hundreds, or even thousands, of wrecked vehicles. Each vehicle sent to a junkyard following an accident represents a tragedy of some degree, with accompanying financial loss, human injury, and perhaps loss of life. One of the objectives of the reconstruction of vehicle accidents is to tell a vital part of that story. Automotive accident reconstruction is the process of determining what happened to the vehicles and persons involved in an accident and how it happened, using the information available after the accident occurred. This task must produce results that are reasonably accurate. Reconstruction is a procedure carried out with the specific purpose of estimating in both a qualitative and quantitative manner how an accident occurred using engineering, scientific, and mathematical principles based on evidence obtained through an accident investigation.

The collection of facts associated with the circumstances of an accident is referred to as accident investigation. Determining what happened and how it happened is usually referred to as accident reconstruction.

A third facet of postaccident analysis attempts to answer the question of why the accident occurred (causation or fault). This is almost always of interest to the various parties involved. Since vehicles are, or should be, controlled by humans, answering the why question more often than not involves motivation and human psychology. Legal issues can also arise in these incidents. These issues include the violation of criminal and traffic laws where the question of fault is placed before a jury. In this book, the topics of accident investigation, human factors, causation, and fault are not explicitly addressed. Often the effects of these topics are interrelated, and this interrelationship must be addressed to some extent. Indeed, it would be rare to be able to carry out a good reconstruction with little or no physical evidence and data. This book concentrates on reconstruction—the determination of how
an accident happened. That said, it is sometimes necessary to combine investigation and reconstruction. Often a reconstructionist may recognize a need for information not gathered initially and must obtain it later. An accurate reconstruction cannot be carried out without a good investigation.

A number of books already exist on the topic of accident reconstruction. With some notable exceptions, many of them are tomes devoted to how the authors and perhaps a few colleagues used intuition and insight to decide how they thought an accident happened. In a few cases, these books are collections of “war stories” or case histories, usually presentations of one view of the events. In contrast, this book is one of methods. The perspective taken here is that accident reconstruction is a field of applied science, namely an application of the principles of science, mathematics, and engineering; accident reconstruction is a quantitative endeavor. The same principles of mathematics, physics, and engineering that allow us to safely race vehicles more than 200 mph, build space stations, and navigate the depths of the oceans can be used to reconstruct vehicle accidents. This requires that reconstructions not only be based on the physical evidence and information gathered from an accident investigation but also be based on the laws of nature. A concerted goal of this book is to raise the analytical level of accident reconstruction practice such that commonly known scientific, engineering, and mathematical methods increasingly become a more common part of the field. During the preparation of this book, the authors have assumed that the readers and users of this book and accompanying software (available separately from the authors) have a proper educational background and experience to fully comprehend the material.

The United States is not the only country with junkyards (repositories of disadvantaged vehicles, to be politically correct). Vehicle accidents, of course, happen all over the world. Fortunately for engineers, and everyone else for that matter, the laws of mathematics and physics are the same everywhere. Consequently, applications of the material contained herein are not limited by geography. Applications using international units do arise. Therefore, dual units, customary U.S. units (based on the units of foot, pound force, and second) and the metric system (based on the meter, Newton, and second) are used throughout the book. Certain items are avoided, such as the confusion between mass and weight. In customary U.S. units, mass sometimes is given the unit of slug. Its use is avoided here. One slug is equivalent to 1 lb·s²/ft; all appearances of the unit pound, abbreviated as lb, in this book refer to units of force. Similarly, in the metric system, a kilogram is considered to be a unit of mass and the corresponding unit of force is the Newton. The practice of stating weight (a force) in kilograms can cause confusion. This is avoided by always using units of Newtons for weight and kilograms for mass. The convention followed here is that weight is the measure of the force of gravity, and the metric unit of force is the Newton, abbreviated as N.

A summary of the topical coverage of the book is not given here. The reader can simply look at the table of contents to see the list of topics. Two features of this book are unique, however, and deserve some mention. One is the coverage of the topic of uncertainty, and the other is the use of many examples throughout the book. Many analysis and reconstruction methods can be implemented using spreadsheet technology. This has been done by the authors, and solutions to the examples are in the form of input information and results
preface to the first edition

printed directly from computer output. Tools contained in popular spreadsheets often allow analysis techniques (time forward computations) to be used for reconstructions (time reverse computations).

A common omission made by all accident reconstructionists at one time or another is to measure something or make a calculation based on a measured or estimated parameter and come up with the answer, look at it, and if it seems to make sense, present it as a definite result or finding. For example, we say “that car was going 25 mph (36.7 ft/s, 11.2 m/s).” But could it have been 26.2 mph or 24.5 mph? How certain are we of the result? How certain can we be of the result? These questions refer to the uncertainty associated with a measurement or calculation based on a measurement, a group of measurements, or a group of calculations. A view is taken here that we actually are estimating values of dynamical variables. Some estimates have high accuracy and some not so high. The uncertainty associated with results based on these estimates of dynamic variables should always be considered. Determining uncertainty is not always easy to do—but difficulty is not a reason for omitting it. The topic of uncertainty is the first technical topic covered in this book. The uncertainty of a reconstruction may be difficult to calculate, but the authors hope that users of this book will appreciate that a reconstructed speed of a vehicle presented with five significant digits of precision has limited accuracy when the only available skid mark length used in the reconstruction calculation was measured by pacing off the distance.

As already mentioned, a distinction is drawn here between accident reconstruction and accident investigation. The latter is considered to be the process of gathering physical and testimonial evidence from an accident scene, vehicles, and eyewitnesses. It is considered as a field of its own. Investigation is most often executed by police officers and sometimes by insurance investigators. As any other human endeavor, it can be done well and can be done poorly. Several institutes exist across the country, such as the Northwestern University Traffic Institute, Texas Transportation Institute, Institute of Police Technology and Management (University of North Florida), and others, for training investigators to standardize and improve investigation practices. Although there is a need for each to know what the other does and there is an overlap in knowledge and tasks, a trained accident investigator is not the same as an accident reconstructionist, just as an accident reconstructionist is not a trained accident investigator.

Different aspects of an accident reconstruction frequently are segregated into the categories of human, vehicle, and environment. The study of human performance and behavior as it relates to vehicular accidents belongs to the study of human factors. This topic is not covered here. Another important aspect of accident reconstruction involving humans is that of occupant kinetics, kinematics, and biomechanics—the study of the motion of vehicle occupants and the physical interaction of a body with interior surfaces and restraints. These concepts are not covered. Environmental topics include such things as the design and performance of roadways, poles and barriers, signs, traffic signals, and their interaction with accidents and crashes. These topics are not covered. As in all professions, the work of accident reconstructionists involves communication and reporting of results. Though they can be extremely important, report writing and diagram preparation are not covered. Other
topics omitted include those of finite element analysis of vehicle crash deformation and
dynamical crush simulation, such as Simulation Model of Automobile Collisions (SMAC),
Simulation Model Nonlinear (SIMON), and others.

Collectively, the authors of this book have over 45 years of experience in the practice of
vehicle accident reconstruction as well as with the research associated with accident
reconstruction methods. Based on this experience, the topics covered throughout the 11
chapters and appendices should provide the methods to quantitatively reconstruct the
vast majority of vehicular accidents. Not all accidents involve a crash, or collision, of two
vehicles, but most do. Planar impact mechanics (Chapters 6 and 7) is used extensively in
the reconstruction of crashes, often combined with estimation of crush energy (Chapter 8).
Evidence from the motion of vehicles before an impact or following an impact, or both, often
supplies vital information to a reconstruction. Vehicle dynamics simulation (Chapter 11) is
invaluable in modeling such motion. Simulation of vehicle dynamics requires the knowledge
of how tire forces are generated (Chapter 2), a topic that all accident reconstructionists must
thoroughly understand. Methods for the analysis of accidents involving a single vehicle,
such as rollovers, pedestrians and bicycle riders hit by cars, or simply yaw marks made by a
single vehicle during a sudden high-speed turn, are covered individually.

Each accident reconstruction is unique as no two accidents are the same. Moreover, the
reconstruction of these accidents can also require the use of different methodologies because
of variations in physical evidence and investigative information. This leaves plenty of room
for ingenuity and insight for the application of the methods presented in this book. The
authors hope this book is useful to those who want to find out how accidents occurred.
With the second edition, as with the first, the authors benefited from many discussions with the members of the Accident Investigation and Reconstruction Practices (AIRP) Committee of the SAE and the attendees, authors, and presenters at the annual Accident Reconstruction sessions held at the SAE World Congress. The annual pilgrimage to Detroit continues to provide a fertile environment for improved understanding and advancement of the many technical subjects that form the basis of the methods used in the field of accident reconstruction.

This enhanced second edition benefits from the numerous times that the authors presented material from the first edition while teaching the SAE Accident Reconstruction Methods seminar from 2004–2010. Many students who attended the class asked insightful questions and made numerous interesting suggestions regarding the material. These questions and suggestions led to many improvements in the material presented in the class. These improvements have been incorporated into this second edition.

The efforts by Kevin Manogue in organizing the original manuscript and preparing examples, and by John McManus, Alan Asay, Don Parker, and Jim Sprague in reading some or all of the original manuscript were followed by Matt Londergan and Weimin Yue who each read part of this second edition. Special thanks goes to Linda Trego for her many helpful comments and changes that improved the manuscript.

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