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Finite Element Analysis for Design Engineers

Second Edition

Paul M. Kurowski
Acknowledgements

This book is dedicated to my wife Elzbieta Kurowska for her encouragement and support.

Paul M. Kurowski
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Preface

During 60+ years of its development the Finite Element Analysis evolved from an exotic analysis method accessible only to specialized analysts into a mainstream engineering tool. Phenomenal progress in computer hardware and operating systems combined with same progress in Computer Aided Design made the Finite Element Analysis available to design engineers to use as a design tool during product design process.

Many books have been written about the Finite Element Analysis. At one end of the spectrum we find books going very deep into theory and at the other end of the spectrum, software manuals explaining how to use certain FEA program. There is little FEA literature taking “middle ground” approach and specifically addressing the needs of design engineers who use the FEA as an everyday design tool. This book attempts to fill this void by focusing on understanding of FEA fundamentals which are explained by simple, intuitive examples understandable by any mechanical engineer. “Finite Element Analysis for Design Engineers” takes practical approach, characteristic to the attitudes of design engineers, and offers the readers an opportunity to try out all discussed topics by solving downloadable exercises using their own FEA program.

Finite Element Analysis for Design Engineers is a very broad field of knowledge. It is difficult to write a book in a “linear” fashion; repetition in discussing concepts, techniques and methods can’t be avoided. For this reason, some topics are discussed more than once taking advantage of a growing body of knowledge as reader progresses through the book.