Several outstanding text and reference books on road vehicle dynamics merit consultation for those readers who wish to pursue these topics further. Also, several publications are devoted to presenting research results and in-depth case studies in road vehicle dynamics. The following list is only a representative sample of the many excellent references that include journals and periodicals on road vehicle dynamics.

**Books**


**Journals and Periodicals**

**General Journals**

Most of the information on road vehicle dynamics is found in papers published in journals or presented at conferences. Relevant papers can be found in journals covering mechanical systems, dynamics, or mechanical engineering in general (e.g., *Dynamics & Control*, Kluwer Academic Publication, Dordrecht, The Netherlands; *Journal of Guidance, Control and Dynamics*, American Institute of Aeronautics and Astronautics (AIAA), New York, USA; and several journals of the American Society of Mechanical Engineers (ASME) and other similar societies).

**Specialized Journals on Road Vehicle Dynamics**

SAE International publishes specialized journals and conference proceedings, organizes conferences, and promotes road vehicle dynamics in numerous ways. The following are specialized journals on road vehicle dynamics:

1. *Accident Reconstruction Journal* (ARJ), Waldorf, MD.
2. *Associazione Tecnica dell’Automobile* (ATA), Torino, Italy.
3. *Automobiltechnische Zeitschrift* (ATZ), Berlin, Germany.
Index

Acceleration, 471–528
  and damping, 306t
  on horizontal road, 473–475
  lateral, 317–318
    instantaneous, 579–580
    normalized, 327–333
  load transfer during, 473–480
  optimal, under steering, 550–556
  power-limited, 471, 499–526
  and steering, 548–556
  traction-limited, 471, 480–499
Accident reconstruction, 563–628
  formulae for, 789–804
    360-degree momentum speed analysis, 797–798
    center of mass, 789–792
    combined speeds, 795–796
    critical curve speed, 794–795
    fall speed, 802–803
    flip speed, 803–804
    kinetic energy and speed, 801–802
    rollover speed, 799–800
    sideslip, 794–795
    slide-to-a-stop speed, 792–794
    tip speed, 799–800
    vault speed, 803–804
    weight shift and speed, 800–801
    yaw, 794–795
  software for, 608–623
 CRASHEX, 611

Engineering Dynamics Corporation (EDC), 613–615
Expert AutoStats, 622
Macinnis Engineering Associates (MEA), 615–616
Maine Computer Group, 616–617
McHenry Software, Inc., 617–620
REC-TEC with DRIVE$^3$ and MSMAC$^R_T$, 608–609
VCRware, 609–611
VDANL, 620–622

Accidents
distribution rate, 346t
rollover, 347t
See also Fatalities

Active control, 298–302
rollover, 350–352
Active controlled mount (ACM), 301
Actuation force, in braking, 429t
Adhesion coefficient
  longitudinal, 191–193
  road, 193

Adjoin matrix, 646

Aerial photographs, in accident reconstruction, 575–577
Aerodynamic forces, 436–437, 476
Airborne phase, 341
Algebraic equations, 660–664
Aligning torque, 179, 198, 199f, 201–204, 396
All-wheel drive, 480–482, 486–488
Almost periodic data, 95, 96f
Amplitude
and frequency, 140f
peak, 107–108
Analytical dynamics, 59
Angles, direction, 638
Angular acceleration, 34–37
Angular momentum, 38, 48–51
of rigid body, 39–40
Animations, in accident reconstruction, 576
Anthropomorphic test dummies (ATDs), 342
Anti-lock braking system (ABS), 459–460, 462–463
Anti-roll-bar, active, 351–352
Anti-rollover braking (ARB), 352
Approximate methods for nonlinear systems, 128–141
Asymmetrical matrix, 645
Augmented matrix, 673
Autocorrelation function, 152–153, 164f
Automated manual transmission (AMT), 520
Automatic transmissions, 514–517
Axis system, 179f
Axle plane, 253
Axles, limited slip, 526–527
Banded matrix, 647
Bicycle model, 361
Bilateral constraints, 63
Body mode, 265
Body response, vehicle
and road excitation, 266–269
and vehicle excitation, 271–272, 273f
Body side bracket, 261
Bounce, 464
Bounce center, 279–281
Bounce motion, 240, 241f
Bounce-pitch model, 232, 274–281
Bracket design, 251–252
Brake assist, 464–465
Brake by wire, 466
Brake squeal, 465
Brakes
disk, 416
drum, 416
torque distribution in, 419, 421–428
engine, 416
exhaust, 416
torque distribution in, 419–431
types of, 416–417
Braking, 415–467
anti-lock braking system (ABS), 459–460, 462–463
anti-rollover, 352
of both axles, 444–450
of front axle, 441
on horizontal road, 432–433
load transfer during, 431–441
lock-up, 443–444
front, 445–446
rear, 446–447
optimal performance, 441–459
under steering, 542–548
of rear axle, 441–443
recent advances in, 464–466
safety considerations, 443–444, 459–465
of single axle, 441–444
and skid, 460–462
and steering, 536–542
and steering, on horizontal road, 537
temperature in, 429–431
tests of, 535
Braking distance, 417–419
Braking force, 420, 450–458
Braking maneuver, 192
Camber angle, 208, 209f, 210f
effects of change in, 392–394
Camber thrust, 208, 209f
Cambered tire models, 209–210
with roll elastic deformation, 211
Camshaft, 13, 14f
Cardan universal joint, 256f
Cartesian coordinate system, 33f, 60f
Case studies, 772–777
Center bearing plane, 253
Center of gravity, 254f, 260, 341f
Center of mass, 789–792
Central difference method, 757–758, 763–765
Centrifugal force, 254f
Centroidal mass moment, 51
Characteristic equation, 99
Chassis system effects on handling characteristics, 385–397
Closed-loop control, 298f, 303f
Coefficients,
damping, 289t
of friction, 568–569, 570t
in braking, 429t
of longitudinal adhesion, 191–193
of road adhesion, 193
of rolling resistance, 188t
roughness, of road profiles, 235t
stiffness, 289t
Cofactor, 646
Column matrix, 644
Combined speeds, 795–796
Combined throw distance model, 605, 606
Complete solution, for motion of mass, 108–109
Complex frequency response function, 104
Complex matrix, 645
Complex multiplication, 694–695
Complex periodic data, 94–95
Complex shifting, 694
Complex vectors, 635–636
Computer simulations, 7
Configuration, 4
Conjugate matrix, 648
Conservation
  of energy, 11, 41–42, 573
  of momentum, 10
Conservative force, 41
Constant radius turn, 404–405
Constant speed test, 406
Constant steer angle test, 406–408
Constraint equations, 5
Constraints, 4–7, 61–63
  geometric, 11
  workless, 64–66
Contact area, forces and torques in, 360–361
Contact forces, 360
  deformation of, 215f
Continuous systems, 3, 10
Continuously variable transmissions (CVTs), 519–520
Control
  active, 298–302
  basic concepts, 298–299
  semi-active, 302–306
Control force, 299f
Controllability, tests of, 534
Conversion, of units, 785–788
Convolution, real, 694–695
Coordinate transformation, 60
Cornering, 409–410
  lateral force transfer effects on, 386–391
  severe, and rollover, 312
  tests of, 535
  tire mechanics model considering, 211–227
  tractive force effects on, 396–397
Cornering characteristic, 196–198, 221f
  mathematical model of, 198–207
  with lateral bending deformation of tire
  case, 204–207
Cornering forces, 358–359
Coulomb damping, 27–29
  modeling of, 28–29
Cramer’s rule, 660–663
Crank mechanism, 35f
CRASHEX, 611
Crest value, 285
Critical damping constant, 101–102
Critical sliding velocity, 340–341
Critical speed, 556–558, 588
  curve, 794–795
Cross-correlation function, 166
Crush energy, 594–595
Curvature response, 385
Customer satisfaction, 231
Customers
  evaluations by, 531–536
  target, 532–533
Cylinder, 54f
  pressure variation in, 239
D'Alembert's principle, 11, 68–69
Dailey formulae, 582–583
Damped systems, multiple-degrees-of-freedom, 115–117
Damping, 12, 302, 372–375
  and acceleration, 306t
  coefficients of, 289t
  combining elements of, 30–32
  Coulomb, 27–29
    modeling of, 28–29
  dry friction, 27–29
  elements of, 25–32
  hysteretic, 29
  material, 29
  solid, 29
  structural, 29
  viscous, 25–27
    modeling of, 26–27
Damping characteristics, 180
Damping ratio, 267–268, 272
Dashpots, 25
  in parallel, 30–31
  in series, 31–32
Deceleration, 417–419, 420t
Deflection
  suspension, contribution to rollover, 324–325
  tire, contribution to rollover, 323–324
Deformation, roll elastic, 211
Degrees of freedom, 4–9, 59–60
Determinants, 652–657
Deterministic data, classification of, 92–97
Deterministic process, 147
Diagonal matrix, 644
Diesel engines, 502–507
Differential, torque multiplying, 526
Road Vehicle Dynamics

Differential equations
of motion, 53–54, 83–85, 127–128
solution of, 701–702
Differentiation, 674, 689–690
Dirac delta function, 158, 686–687
Direction angles, 638
Discomfort evaluation, 290–298
Discrete models, 7
Discrete parameter system, 13–14
Discrete random variable, 147
Discrete systems, 3, 4, 10
Disk, 65f
vertical, 62f
Disk brakes, 416
torque distribution in, 428–429
Displacements, 7
lateral, 581
longitudinal, 580
Dissipation function, Rayleigh’s, 78–79
Distributed parameter model, 4
Drag, 476
Drag effects, 436–437
Drag factors, 568–571
DRIVE3, 608–609
Driveline modes, 257
Drivelines
imbalance forces, 253–255
vibrations from, 252–261
Drivers
evaluations by, 531–536
perception and response of, 573–575
Driveshaft modes, 257
Driveshafts, 253–255
Drivetrain configurations, 480–483
Driving maneuver, 192–193
Drum brakes, 416
torque distribution in, 419, 421–428
Dry friction damping, 27
Duffing’s equation, 128
Dynamic constant steer angle test, 406–408
Dynamic handling characteristics, 372–385
Dynamic matrix, 110
Dynamic response at random input, 272–274
Dynamic rollover model, 333–338
for dependent suspension, 334–336
for independent suspension, 336
Dynamic rollover threshold, 338–341
Dynamic stability index, 338–339
Dynamic systems, 3
analysis of, 91–174
classification of models of, 4, 5f
linear, 97–123
nonlinear, 123–147
Dynamics
analytical, 59
of handling, 357–413
of ride, 231–307
of tires, 177–227
lateral, 196–211
longitudinal, 186–195
review of, 32–59
vehicle
application of PSD to, 166–168
tests and evaluations, 533–536
total, 531–560
East-west configuration, 240
Effective force, 69
Eigenvalues, 412, 413f, 664–668
damped system, 115–117
properties of, 667–668
undamped system, 110–115
Eigenvectors, 664–668
damped system, 115–117
properties of, 667–668
undamped system, 110–115
Eight-degrees-of-freedom system model, 400–403
Elastic elements, 10
Electric motor, 501
Electric retarders, 417
Electric vehicles, 465–466
Electro-rheological (ER) fluid, 303–304
Energy
conservation of, 11, 41–42, 573
crush, 594–595
kinetic, 82, 572
potential, 83, 572–573
principle of, 40–41
Engine brakes, 416
Engine mount, control system for, 299f
Engine vibration, 232
Engineering Dynamics Corporation (EDC), 613–615
Engineering models, in accident reconstruction, 575–576
Engines, 501–507
diesel, 502–507
four steps of working processing, 242f
future, 503–504
gas turbine, 501
gasoline, 502, 504–507
hybrid, 503, 504f
internal combustion, 502–508
mount isolation system, 243
Index

Ranking vapor cycle, 502
Stirling cycle, 501–502
vibration sources from firing pulsation, 242–244

Enveloping characteristics of tires, 185–186

Equations
algebraic, 660–664
characteristic, 99
costant, 5
damped iterative method, 129–131
Duffing’s, 128
equilibrium, 67–68, 755
governing, 10–11
Lagrange’s 59–67, 74, 79–80
of motion, 81, 125–127, 564, 565r
using D’Alembert’s principle, 69–71
differential, 53–54, 83–85, 127–128
Lagrange’s, 74
linearized, 55–57
for rigid body, 39
undamped iterative method, 128–129
Equilibrium equations, 67–68, 755
Equivalent springs, 18–21, 23–25
Equivalent system, 9f
Evaluations, by drivers, 531–536
Exact solutions, 124
Exhaust acoustic mode, 261
Exhaust brakes, 416
Exhaust side bracket, 261
Exhaust systems, vibrations from, 257–261
Expert AutoStats, 622
Explicit schemes, 763–766
Exponential function, 682

Fall speed, 802–803
estimates for, 584–586
Fatalities
rollover, 347f
in single-vehicle crashes, 346f
by vehicle body type, 347f
Final value theorem, 693
Finite difference method, 756–757
Finite element analysis, 258, 281–282
Finite element model, 343
Firing order, 243, 244f
Fishhook maneuver, 337, 338f
Fixed contact patch model of tires, 182–183
Flex decoupler, 258
Flip speed, 803–804
estimates for, 587
Floor vibration, 302f
Forced vibrations, 92

of single-degree-of-freedom systems, 102–107
for multiple-degrees-of-freedom systems, 119–120

Forces
actuation, in braking, 429f
aerodynamic, 436–437, 476
braking, 420r, 450–458
centrifugal, 254f
conservative, 41
contact, 360f
control, 299f
cornering, 196, 201–204, 358–359
effective, 69
frictional, 27, 78
generalized, 71–74, 75
hanger, 259, 260f
imbalance, 253–255
inertial, 2
imbalance, 239
powerplant, 244
lateral, 177–179
transfer effects on cornering, 386–391
lift, 2
longitudinal, 179, 194–195
nonviscous, 78
normal, 179
periodic, 105–106
roll, 211
side, of tires, 208f
slip, 196
spring, 15–16
on tires, 177–180
tractive, 179
effects on cornering, 396–397
vertical, on rolling tire, 200f
viscous, 78

Forcing function, 158, 159f, 160
Fore-aft motion, 240, 241f
Forward projection trajectory model, 605, 606
Four-by-four systems, 480–482, 486–488
Four-degrees-of-freedom model, half car, 300f
Four-degrees-of-freedom systems, torsional, 84f
Fourier analysis, 154–156
Fourier integral, 155–156
Fourier series, 154–155
Fourier transform, 156–157
Free vibrations, 91
of single-degree-of-freedom systems, 98–101
Frequency composition, 163
Frequency of oscillation, 54
Frequency response, 380–384
Frequency response function, 167
Frequency response method, 160–161
Friction coefficients, 568–569, 570r, in braking, 429r
Friction force, 27, 78
Friction rollover, 313
Front axle, braking of, 441
Front lock-up, 544–545
Front skid, 486–487, 551–552
Front suspension deflection, 301f
Front-wheel drive, 480–484 and manual transmissions, 510, 512–514
Funk-Cormier-Bain model, 624–625
Future engines, 503–504

Gas turbine engines, 501
Gasoline engines, 502, 504–507
Gaussian probability density function, 154f
Gaussian random process, 153
Gear meshing, 255–256
Gear ratios, 513f
Gear train, 52f
Geared systems, 52–53, 58f
Gears, transmission error, 255–256
Generalized coordinates, 4–7, 9, 59, 60
Generalized force, 71–74, 75
Generalized mass, 110
Geometric constraints, 11
Gerodisc, 526
C.G. Gim theoretical model, 212–214
Glossary, 705–753
of matrix terms, 676–678
Governing equations, 10–11
Grade, effect on load transfer, 438–441, 477–480
Graphical method, 142–146
Ground-based controlled rollover impact system (GB-CRIS), 342, 343f
Ground-impact phase, 341
K.H. Guo tire model, 214–223

Handling characteristics
chassis system effects on, 385–397
dynamic, 372–385
overturning limit, 397–399
steady state, 365–372
testing of, 403–413
Handling dynamics, 357–413
nonlinear models of, 399–403
Handling parameters, 412
Handling safety, 397–399
Handling tests, 535

Hanger forces, 259, 260f
Hardening characteristic, 139f
Heading angle, 581
Holonomic constraints, 61, 67
Holonomic systems, 6, 74–76
Houbolt method, 766–768
Human body, 282–283
acceleration sensitivity curves, 285f
models of, 290–298
linear, 294–295
nonlinear, 295–298
in rollover, 341–345
vibrations transferred to, 242
Hybrid engines, 503, 504f
Hybrid III ATD, 342
Hybrid vehicle, 465–466
Hydraulic retarders, 416–417
Hysteric damping, 29

Imbalance force, 253–255
Impact analysis, 591–598
noncentral, 593–594
straight central, 591–593
Implicit schemes, 766–772
Impulse, 42–44
Impulse function, 685–686
Impulse input excitation response, 379–380
Impulse response method, 158–160
Indefinite matrix, 671
Inertial forces, 2
elements of, 25
imbalance, 239
powerplant, 244
Initial value theorem, 693
Input-output relationship, 106f
Integration, 674, 690–691
direct numerical methods, 755–777
Internal combustion engines, 502–508
International System of Units, 781–788
Inverse Laplace transforms, 696–700
Inverted pendulum, 55f
ISO 2631 (Frequency Weighting to Vibration), 291
Isolaters, 298
Isolation systems, single-degree-of-freedom, 245f, 246f
Iterative method
damped equation, 129–131
undamped equation, 128–129
Joint probability density function, 165
Index

Kinematics, of rigid bodies, 33–34
Kinetic energy, 82, 572
and speed, 801–802

Lagrange’s equation, 59–67, 74, 79–80
Lane change maneuver model, 577–584
Laplace transforms, 679–702, 691–693r
inverse, 696–700
properties of, 681–682, 695–696r
Large mass method, 260
Lateral acceleration, 317–318
instantaneous, 579–580
normalized, 327–333
Lateral displacement, 580
Lateral dynamics of tires, 196–211
Lateral force compliance steer, 395–396
Lateral forces, 179
transfer effects on cornering, 386–391
Lateral motion, 240, 241f
Lift forces, 2
Limited slip axle, 526–527
Limpert formulae, 581–582
Lindsted-Poincare method, 137
Linear dependence, 636–637
Linear dynamic systems, 97–123
integration methods for, 772–774
Linear human body model, 294–295
Linear models, advantages of, 7
Linear momentum, 573
collection of, 37
Linear quadratic regular (LQR) control, 301
Linear seat modeling and transmissibility, 286
Linear springs, 22–23
potential energy of, 18
Linear systems, 4
multiple-degrees-of-freedom, 109–110
single-degree-of-freedom, 98, 126f
Load distribution function, 203–204
Load transfer
during acceleration, 473–480
during braking, 431–441
on horizontal plane, 437, 438f
Lock-up, 546–548, 558–559
front, 544–545
rear, 545–546
Logarithmic decrement, 101–102
Longitudinal adhesion coefficient, 191–193
Longitudinal displacement, 580
Longitudinal dynamics of tires, 186–195
Longitudinal force, 179
tire, theoretical model of, 194–195
Longitudinal slip, 221f
tire mechanics model considering, 211–227

Longitudinal vibration, 17f
Lumped inertia model, 58–59
Lumped parameter model, 4
Lumped parameter system, 13–14
Lumped systems, 3, 4, 10

M-crash, 618
M-edit, 618–619
[m]-orthogonal eigenvector, 120–122
[m]-orthonormal modal matrix, 117–119
m-smac, 617
m-vhosm, 617–618
Macinnis Engineering Associates (MEA), 615–616
MADYMO ATD, 342, 343, 344f
Magic formula model, H.B. Pacejka, 224–227
Magneto-rheological (MR) fluid, 305
Maine Computer Group, 616–617
Manual transmissions, 509–510

Mass, 12
center of, 789–792
elements of, 25
Mass ratio, 268, 272
Material damping, 29
Mathematical models, 4, 5f, 10
Matrices
adjoin, 646
asymmetrical, 645
augmented, 673
banded, 647
column, 644
complex, 645
conjugate, 648
definitions of, 643–648
diagonal, 644
indefinite, 671
negative definite, 670–671
non-singular, 647
null, 644
partitioning of, 672–672
positive definite, 668–670
rank of, 647
real, 644
row, 644
singular, 647
square, 644
symmetrical, 645
transpose of, 645
triangular, 646
tridiagonal, 647
unit, 644
zero, 644

Matrix analysis, 643–675
Matrix calculus, 674–675
Matrix operations, 648–651
Mechanical systems, 45–46
Minor, 646
Mixed product, 639
Modal decoupling, 249–250
Modal energy distribution, 250–251
Modal frequencies, 250–251
Models
automobile, simplified, 13f
bounce-pitch, 232, 274–281
classification of, 4, 5f
distributed parameter, 4
eight-degrees-of-freedom, 400–403
eight-degrees-of-freedom, 300f
Funk-Cormier-Bain, 624–625
C.G. Gim theoretical, 212–214
half car, 300f, 301f
human body, 290–298
linear, 294–295
nonlinear, 295–298
lane change maneuver, 577–584
linear, advantages of, 7
lumped parameter, 4
mathematical, 4, 5f
multi-body facet, 343–344
multiple-degrees-of-freedom, 399–400
multiple-degrees-of-freedom, 399–400
nonlinear, of handling dynamics, 399–403
nonstationary, 4
H.B. Pacejka magic formula, 224–227
for pedestrian–vehicle collisions, 601–608
pitch plane, 464
plane, 281
yaw, 357, 358f, 361–365
quarter car, 232, 262–274
rollover
dynamic, 333–338
rigid body, 316–322
rigid dynamic, 333–334
suspended vehicle, 321–333
stationary, 4
stochastic, 4
three-dimensional, 281, 282f
time-invariant, 4
time-variant, 4
tire
cambered, 209–211
K.H. Guo, 214–223
fixed contact patch, 182–183
longitudinal force of, 194–195
longitudinal slip and cornering, 211–227
nonsteady-state semi-empirical, 219–223
point contact, 181–182, 186f
steady-state simplified theoretical,
214–219
time-varying contact patch, 183–185
vertical vibration mechanics, 181–185
vehicle ride, 261–282
virtual vehicle, 337, 338f
wrap trajectory, 605, 606
Moments
aligning torque, 179
of inertia, 50–51
overturning, 179
rolling resistance, 179
Momentum
angular, 38–40
conservation of, 10
linear, 573
conservation of, 37
principle of, 42–44
Momentum couple, 43, 44f
Mount and bracket system, 251f
MSMAC RT, 609
Multi-body facet model, 343–344
Multiple-degrees-of-freedom systems, 146–147
damped, 115–117
direct numerical integration methods for,
762–763
forced vibration solution of, 119–120
linear, 109–110
models for, 399–400
response to random inputs, 170–173
undamped, 110–115
Multiplication, complex, 694–695
NADSdyna, 408
Natural frequencies, 121, 122–123, 247–248,
372–375
damped, 101–102
Negative definite matrix, 670–671
Neutral steer, 367
Newmark-β method, 771–772
Newton's laws of motion, 32, 564–565
  second law, 11, 46, 47
Noise, vibration, and harshness (NVH), 259
Nomenclature, 811–832
Nondeterministic process, 147
Nonholonomic constraints, 62, 67
Nonholonomic systems, 76–78
Nonlinear dynamic systems, integration methods for, 774–777
Nonlinear models of handling dynamics, 399–403
Nonlinear seat–human body modeling, 295–298
Nonlinear seat modeling and transmissibility, 286–289
Nonlinear systems, 4
  approximate methods for, 128–141
  characteristics of, 123–124
  dynamic 123–147
  exact methods for, 124–127
  undamped, 135f
Nonperiodic data, 95–97
Non-singular matrix, 647
Nonstationary models, 4
Nonsteady-state semi-empirical tire mechanics model, 219–223
Nonviscous forces, 78
Normal force, 179
Normal mode summation method, 120
Normal random process, 153
Normalized lateral acceleration, 327–333
North-south configuration, 240
Null matrix, 644

Occupant, in rollover, 341–345
Open-loop control, 298f
Orthogonality condition, 117
Oscillation, 132–134
  frequency of, 54
Oscillation center, 278
Overhead valve, 14f
Oversteer, 368
Overturning limit handling characteristics, 397–399
Overturning moment, 179

H.B. Pacejka magic formula model, 224–227
Parameter method, variation of, 140–141
Parameters, 4
Parking, tests of, 534
Parseval's theorem
  for nonperiodic functions, 156
  for periodic functions, 155
Partial fraction expansions, 696–701
  when Q(s) has complex conjugate roots, 698–699
  when Q(s) has distinct roots, 697–698
  when Q(s) has repeated roots, 699–700
Partitioning of matrices, 672–672
Peak amplitude, 107–108
Pedestrian–vehicle collisions, 598–608
  empirical models for, 603–605
  mathematical and hybrid models for, 601–608
Pell's method, 144–146
Pendulum, 132f, 80f
  inverted, 55f
Pendulum mounting system, 248f
Performance, tests of, 536
Periodic data
  almost periodic, 95, 96f
  complex, 94–95
  sinusoidal, 93
Periodic force, 105–106
Perturbation method, 134–140
Phase plane representation, 142
Phase velocity, 142–143
Physical system, mathematical modeling of, 10
Pitch, bounce-pitch model, 232, 274–281
Pitch center, 279–281
Pitch motion, 240, 241f
Pitch plane models, 464
Plane model, 281
  yaw, two-degrees-of-freedom, 361–365
Plane motion, 43
Plate clutch, 526
Pneumatic trail, 177, 197, 396
Point contact model of tires, 181–182, 186f
Polar coordinate system, 60f
Positive definite matrix, 668–670
Potential energy, 83, 572–573
Power-limited acceleration, 471, 499–526
Power losses, 504–505, 521
Power spectral density
  in spatial frequency, 233–236
  in temporal frequency, 237–239
Power spectral density function, 163–164
  application to vehicle dynamics, 166–168
Powerplants, 499–501
  inertia forces and moments of, 244
  isolation design for, 244–248
  modeling with exhaust system, 258–259
  mounting systems for
    six-degrees-of-freedom, 249f
    pendulum, 248f
  vibration sources from, 239–252
Powertrain resistance, 437
Pressure, 505–506
variation in cylinder, 239
Principal minor, 646
Probability density function, 149–151, 164f
Gaussian, 154f
Proportional, integral, and derivative (PID)
controller, 300–301
Propulsion, 2
Pulley system, 8f
Pulse function, 684–685

Quadratic forms, 668
Quarter car model, 232, 262–264
dynamic analysis for, 266–274
modal analysis for, 264–266

Radius of turn, ratio of, 370–371
Ramp function, 683–684
Ramp steer input response, 379
Random inputs
dynamic response at, 272–274
multiple-degrees-of-freedom system
response to, 170–173
single-degree-of-freedom system response
to, 168–170
Random process, 148f
Random variable, 147
Rank of a matrix, 647
Ranking vapor cycle engine, 502
Rayleigh’s dissipation function, 78–79
Real convolution, 694–695
Real matrix, 644
Rear axle, braking of, 441–443
Rear lock-up, 545–546
Rear skid, 487–488, 552–553
Rear-wheel drive, 480–482, 484–486
and manual transmissions, 510, 511–512
REC-TEC with DRIVE3 and MSMACRT,
608–609
Recurrence formula, 758
Retarders
electric, 417
hydraulic, 416–417
Rheonomic constraints, 63
Ride comfort, 231–232
Ride discomfort, 290–292
objective evaluation of, 292–293
Ride dynamics, 231–307
Ride models, 261–282
Ride tests, 533–534
Rigid bodies
angular momentum of, 39–40
equations of motion for, 39
kinematics of, 33–34
plane motion of, 43f
rotation of, 240, 241f
Rigid body motion, 6–7
Rigid body rollover model, 316–322
Rigid dynamic rollover model, 333–334
Ritz averaging method, 131–132
Road elevation profile, 234
Road excitation, and body response, 266–269
Road profile, 213–232
Road surface profile, 233, 234f
Rod, rigid, 61f, 65f
Roll angle, 211
Roll elastic deformation, cambered tire model
with, 211
Roll mode, 247–248
Roll motion, 240, 241f
Roll steer, 395
Rolling properties of tires, 208–211
Rolling resistance, 187–188, 436, 476
with toe-in, 188–189
turning, 189–191
Rolling resistance moment, 179
Rolling start, 536
Rollover
active control of, 350–352
analysis of, 311–352
anti-rollover braking, 352
contribution of suspension deflection to,
324–325
contribution of tire deflection to, 323–324
control of, 345–352
definition of, 311–314
friction, 313
GB-CRIS, 342, 343f
importance of, 314
maneuver-induced, 397
models of, dynamic, 333–338
for dependent suspension, 334–336
for independent suspension, 336
occupant in, 341–345
overturning limit, 397–399
research on, 314–315
rigid body model, 316–322
safety control, 350–352
safety, overview of, 345–348
sensing of, 349–350
and severe cornering, 312
and severe steering input, 313
simulation of occupant in, 343–345
simulation tools for, 336–338
Index

843

and speed, 312
steady-state, on flat road, 317, 321–323
suspended vehicle model, 321–333
tripped, 312, 349
untripped, 312, 349
Rollover center, 321, 322
Rollover prevention energy reserve (RPER), 339
Rollover prevention metric (RPM), 340
Rollover propensity, 318
Rollover speed, 799–800
Rollover threshold, 318
dynamic, 338–341
Root mean square value, 165
Roots, complex representation of, 100
Rotational systems, 47–48
Roughness coefficient, 235
Row matrix, 644
RTRD, 408
Runge-Kutta method, 760–762
dynamic, 338–341
fourth-order, 765–766
S.I. System of Units, 781–788
SAE J1995, 506–507
Safety
handling, 397–399
in braking, 443–444, 459–465
requirements, 418–419, 420
and rollover control, 345–352
and transmissions, 526–528
Sample record, 147
Scalar products of vectors, 638
Scleronomic constraints, 63
Seat
evaluation and modeling of, 282–289
and human body modeling, nonlinear, 295–298
and semi-active damper, 304–305
velocity of, 285–286
Seat effectiveness amplitude transmissibility
(SAET), 232, 283–285
Seatbelt, 344
effectiveness in fatality reduction, 348
Second order excitation, 256–267
Secular term, 137, 139
Self-aligning torque, 198
Self-energizing system, 422
Self-excited vibrations, 92
Semi-active control, 302–306
Semi-active damper, 304, 305
Semi-active vibration actuator (SAVA), 302, 303
Shape memory alloys, 305–306
Shift in time, 694
Shock absorbers, 25
Side force of tires, 208
Side pull ratio (SPR), 320–321
Sideslip, partial, 201–204
Sideslip angle, 361–362
Sideswipe collisions, 623–629
modeling of, 625–629
Simulated weight, 319
Simulation of occupant rollover, 343–345
Simulation tools, rollover, 336–338
Simulations, 408
Sines, law of, 35
Single-degree-of-freedom systems
central difference method for, 757–760
direct numerical integration methods for,
756–762
finite difference method for, 756–757
forced vibration of, 102–107
free vibration of, 98–101
general equation for, 756
isolation, 245, 246
linear, 98, 126
response to random inputs, 168–170
Runge-Kutta method for, 760–762
vibrating, 73
response of, 157–161
Singular matrix, 647
Sinusoidal function, 687–688
Sinusoidal periodic data, 93
Six-degrees-of-freedom powerplant mounting
system, 249
Skid, 486–490, 553–556, 566–567
and braking, 460–462
front, 551–552
rear, 552–553
Slide-to-a-stop speed, 792–794
Sideslip, 794–795
Sliding, critical velocity, 340–341
Slip
and braking, 460–462
longitudinal, 221

tire mechanics model considering,
211–227
tires with, 218–219
tires without, 215–218
Slip angle, 196
difference between front and rear, 368–370
Small-amplitude vibration, 97
Softening characteristic, 139
Solid damping, 29
Spatial frequency
power spectral density in, 233–236
relation to temporal frequency, 237
Speed, 233, 566
360-degree momentum analysis, 797–798
Speed (cont.)
and yaw velocity gain, 367f, 368f
combined, 795–796
constant speed test, 406
critical curve, 794–795
critical, 556–558
and damping, 373f
estimates from yaw marks, 588–591
fall, 802–803
estimates for, 584–586
flip, 803–804
estimates for, 587
and kinetic energy, 801–802
rollover, 312, 799–800
slide-to-a-stop, 792–794
tip, 799–800
vault, 803–804
estimates for, 587–588
and weight shift, 800–801
Sphere, 44–45
Spring constant, equivalent, 23–25
Spring force, 15–16
Springs, 8f, 15–18
equivalent, 18–21, 23–25
hardening and softening characteristics, 139f
linear, 22–23
potential energy of, 18
in parallel, 19, 20f, 21
in series, 20, 21f
Square matrix, 644
Stability
analysis of, 384
tests of, 535
vehicle, 558–560
Standard deviation, 151–152, 165
Standing start, 536
Static margin, 371–372
Static stability factor (SSF), 318
Stationary models, 4
Stationary random process, 148
Steady-state handling characteristics, 365–372
Steady-state rollover model for suspended vehicle, 321–323
Steady-state simplified theoretical tire model, 214–219
Steady-state solution, 102
Steer angle, 580
constant steer angle test, 406–408
Steering
and acceleration, 548–556
active, 352
and braking, 536–542
optimal braking under, 542–548
and rollover, 313
tests of, 534–535
Steering system, 391–392, 393f
Steering wheel angle, 580
Step function, 682–683
Step steer input response, 375–379
Stiffness, 12
coefficients of, 289f
equivalent, 21–23
suspension, 327–333
tire, 327–333
vertical, 180, 181f
Stiffness matrix, 110
Stiffness ratio, 269, 272
Stirling-cycle engine, 501–502
Stochastic models, 4
Stopping distance, 417–419, 420f, 575
Structural damping, 29
Support, 2
Suspended vehicle rollover model, 321–333
parameters influencing, 326–333
Suspension
active, 350–351
stiffness, 327–333
virtual model, 337f
Suspension deflection, contribution to rollover, 324–325
Symbols, list of, 811–832
Symmetrical matrix, 645
System response, 7
Temperature, 505–506
in braking, 429–431
Temporal frequency, power spectral density in, 237–239
Temporal mean value, 149
Testing validation, 408
Three-degrees-of-freedom systems, vibrating, 83f
Three-dimensional model, 281, 282f
Tilt table ratio, 318–320
Time-invariant models, 4
Time-variant models, 4
Time-varying contact patch model of tires, 183–185
Tip speed, 799–800
Tire carcass, 211
Tire deflection, contribution to rollover, 323–324
Tire mode, 265
Tires
abraded, 207
braking maneuver, 192
cambered, model for, 209–210
contact patch, 194f, 198f, 200f, 205f
cornering characteristic, 196–198
mathematical model of, 198–207
with lateral bending deformation of tire case, 204–207
cornering forces, 358–359
damping characteristics of, 180
difference between front and rear slip angles, 368–370
distribution of vertical load, 199–200
driving maneuver, 192–193
dynamics of, 177–227 233
lateral, 196–211
longitudinal, 186–195
vertical, 180–186
enveloping characteristics of, 185–186
fixed contact patch model of, 182–183
forces acting on, 177–180
function of, 177
load deflection relationship, 180f
load distribution function, 203–204
load on, 207
longitudinal adhesion coefficient of, 191–193
models of
cambered, 209–211
longitudinal slip and cornering, 211–227
nonsteady-state semi-empirical, 219–223
with slip, 218–219
without slip, 215–218
steady-state simplified theoretical, 214–219
point contact model of, 181–182, 186f
rolling properties of, 208–211
rolling resistance of, 187–188
with toe-in, 188–191
side force of, 208f
skidding, 191
sliding, 191
slipping, 191, 192f
stiffness, 327–333
structure of, 207
theoretical model of longitudinal force, 194–195
time-varying contact patch model of, 183–185
vertical stiffness of, 180, 181f
vertical vibration mechanics models of, 181–185
Torque, 360–361, 504–508
aligning, 179, 198, 199f, 201–204, 396
in braking, 429f
calculation of, 422–428
distribution of, 419, 421–431
Torque converter, 514–519
Torque multiplying differential, 526
Torsional system, 17f
four-degrees-of-freedom, 84f
Trace, 646
Traction control, 527–528
Traction-limited acceleration, 471, 480–499
Tractive effort, optimal, 490–494
Tractive forces, 179, 492–499
effects on cornering, 396–397
Trade-off studies, 533
Trajectory, 142, 143–144, 145f
pedestrian, 599–601
Transfer functions, 58, 106f
between body and seat track, 297
Transfer matrix, 58
Transient data, 95–97
Transient solution, 102
Translational and rotational systems, 48
Translational systems, 46
Transmissibility, 246, 272–274
between body response and road excitation, 266–269
between body response and vehicle excitation, 271–272, 273f
linear seat modeling and, 286
nonlinear seat modeling and, 286–289
of single-degree-of-freedom model, 270
Transmission error, 255–256
Transmission plane, 252–253
Transmissions, 508–520
automated manual, 520
automatic, 514–517
continuously variable, 519–520
manual, 509–510
tests of, 536
Transpose of a matrix, 645
Transverse vibration, 17f
Triangular matrix, 646
Tridiagonal matrix, 647
Tripped rollover, 312, 349
Tripping phase, 341
Turn radius, 580
Turn radius, ratio of, 370–371
Two-degrees-of-freedom systems
vibrating, 81f
yaw plane model, 361–365
Undamped systems
multiple-degrees-of-freedom, 110–115
response of, 101f
Understeer, 367–368
Understeer gradient, 365–368, 404, 410–411
Unilateral constraints, 63
Unit matrix, 644
Units, and conversion, 781–788
Untripped rollover, 312, 349
Vault speed, 803–804
estimates for, 587–588
VCRware, 609–611
VDANL, 620–622
Vector algebra, 635–640
Vector differentiation, rules of, 640
Vector operation, laws of, 636
Vector product, 638
Vector representation, 34
Vectors
complex, 635–636
derivative of, 639–640
norm of, 671–672
scalar products of, 638
three-dimensional, 637–638
Vehicle body response, 261
Vehicle evaluation rating (VER), 233, 290, 291
Vehicle excitation, and body response, 271–272, 273
Vehicle–pedestrian collisions, 598–608
Vehicle ride dynamics, 231–307
Vehicle ride models, 261–282
Vehicle rollover model, suspended, 321–333
parameters influencing, 326–333
Vehicle stability, 558–560
Vehicle system classification, 2, 3
Vehicles, two-wheeled, 162
Velocity, 226
phase, 142–143
Vertical load, tire, distribution of, 199–200
Vertical stiffness of tires, 180, 181
Vertical vibration mechanics models of tires, 181–185
Vibrating systems, 4, 68f, 71f, 127f
elements of, 15–32
single-degree-of-freedom, 73f
response of, 157–161
two-degrees-of-freedom, 83f
Vibration acceleration, and ride discomfort, 292
Vibration analysis, 10–11
Vibration dose values, 285
objective evaluation by, 293
Vibration mechanics models of tires, vertical, 181–185
Vibrations, 3
classification of, 91–92
from driveline sources, 252–261
from engine sources, 232, 242–244
from exhaust system sources, 257–261
floor, 302f
forced, 92
of single-degree-of-freedom systems, 102–107
solution for multiple-degrees-of-freedom systems, 119–120
free, 91
of single-degree-of-freedom systems, 98–101
from internal sources, 239–261
longitudinal, 17f
from powerplant sources, 244, 239–252
random, 147–153
and ride quality, 231
from road sources, 233–239
self-excited, 92
semi-active vibration actuator, 302, 303f
small-amplitude, 97
study of, 85
transverse, 17f
Virtual displacements, 63–64
Virtual vehicle models, 337, 338f
Virtual work
extended to dynamic case, 68–69
principle of, 63–67
Viscoelastic solids, 29
Visco-Lok, 526
Viscous coupling, 526
Viscous damping, 25–27
modeling of, 26–27
Viscous forces, 78
Weight shift and speed, 800–801
Weighted root-mean-square method, 292–293
Wilson-θ method, 768–770
Wind excitation, 231–232
Work, 571–572
principle of, 40–41
Workless constraints, 64–66
Wrap trajectory model, 605, 606
Yaw, 794–795
Yaw motion, 240, 241f
Yaw plane model, 357, 358f
two-degrees-of-freedom, 361–365
Yaw velocity, 376–377
  frequency response of, 380–384
Yaw velocity gain, 365–368

Zero matrix, 644
Rao Dukkipati, Ph.D., P.E., F.A.S.M.E., F.C.S.M.E.
Professor, Chair and Graduate Program Director
Department of Mechanical Engineering
Fairfield University, Fairfield, CT

Rao Dukkipati is a professor and chair of mechanical engineering, and the director of the graduate program at Fairfield University, as well as a consultant in the field of accident reconstruction. He is member of the Connecticut Academy of Sciences and Engineering (CASE) and a Fellow of both the American Society of Mechanical Engineers (ASME) and the Canadian Society for Mechanical Engineers (CSME). Dr. Dukkipati’s other professional memberships include the American Society for Engineering Education (ASEE) and SAE International. He has a Ph.D. in Mechanical Engineering from Oklahoma State University (1973), an M.S. in Mechanical Engineering from both the Andhra University, India (1969) and the University of New Brunswick, Canada (1971), and a B.S. in Mechanical Engineering from Sri Venkateswara University, India (1966). Dr. Dukkipati’s specialized areas of teaching and research include mechanical design, mechanics, vibrations, control systems, engineering system dynamics, economy and reliability, quality control, advanced engineering mathematics, vehicle dynamics, and accident reconstruction. Previously, Dr. Dukkipati worked at Pratt & Whitney Aircraft of Canada of the United Technologies Corporation in Montreal, Canada; the National Research Council of Canada in Ottawa, Canada; the University of Windsor in Windsor,
Canada; and the University of Toledo in Ohio. He is a licensed Professional Engineer in the province of Ontario, Canada. Dr. Dukkipati has published more than 300 papers in national and international journals and conferences and numerous technical reports. He has authored or co-authored more than 30 engineering books, spanning a range of topics that includes vibrations, control systems, engineering system dynamics, economy and reliability, quality control, engineering mathematics, robotics, numerical methods, kinematics and dynamics, railway vehicle dynamics, and road vehicle dynamics. Dr. Dukkipati was awarded the 2008 American Society for Engineering Education (ASEE) New England Section Outstanding Teacher Award.

Jian Pang, Ph.D.
Researcher and PD Engineer
Powertrain Programs
Ford Motor Company, Dearborn, MI

Jian Pang is a senior researcher and PD engineer with Ford Motor Company. He formerly was a technical specialist and senior engineer with Stewart & Steven Service, Inc. in Texas. He received his Ph.D. in Mechanical Engineering from the University of Oklahoma in 1996. He received his B.S. (1985) and M.S. (1991) degrees in Mechanical Engineering from Wuhan University of Technology and Shanghai Jiao Tong University, respectively. Dr. Pang has 20 years of diversified experience in vehicle and ship engineering. He has published more than 40 papers in international and national journals and conferences and more than 50 industrial technical reports in the area of automotive system dynamics, noise and vibration, durability, ship structure, and calculation method analysis. Dr. Pang is the leading co-author of a book on automotive noise, vibration, and harshness. He is a member of the American Society of Mechanical Engineers (ASME) and SAE International. Dr. Pang also has published a novel and more than 100 articles in magazines, newspapers, Internet sites, and so forth.
Mohamad Qatu, Ph.D., P.E.
Adjunct Professor
Oakland University, Rochester, MI
and
Leader, Advanced Research
Ford Motor Company, Dearborn, MI

Mohamad Qatu received his undergraduate engineering degree from Yarmouk University in Jordan (1985) and his M.S. and Ph.D. from Ohio State University in 1986 and 1989, respectively. His areas of specialization include solid mechanics, computer-aided engineering, and noise and vibration. Dr. Qatu’s academic experience includes working as a professor and director of the mechanical engineering technology program at Franklin University, an associate professor of mechanical engineering at Lake Superior State University, a visiting professor at An-Najah National University, and an adjunct professor at Oakland University. Dr. Qatu’s industrial experience includes working for Dresser Industries, Honda of America, Dana Corporation, and Ford Motor Company, where he currently leads a group on noise, vibration, and harshness. He is the author of one book and more than 70 research papers, review articles, and book reviews, and he is a registered Professional Engineer in both Ohio and Michigan. Dr. Qatu has received two patents on noise suppression and several technical awards, and he wrote a few manuals to his credit. Recently, he established a new journal, *International Journal of Vehicle Noise and Vibration*, and serves as its executive editor. Dr. Qatu also is on the editorial board of the *Journal of Composite Structures*. He is a Fellow of the American Society of Mechanical Engineers (ASME) and a member of SAE International.

Gang Sheng, Ph.D.
Research Scientist
Gates Corporation, Rochester Hills, Michigan

Gang Sheng received his undergraduate engineering degree and his M.S. from Shanghai Jiao Tong University in 1984 and 1987, respectively, and his Ph.D. from Singapore Nanyang Technological University in 1997. Presently, he is an assistant professor at the University of Alaska, Fairbanks. Previously, Dr. Sheng worked as a research scientist
at Gates Corporation, an advisory scientist at IBM, a principal consultant for CMA in the United States, a principal engineer at Singapore National Data Storage Institute, a principal engineer/group leader at Sony in Singapore, and an assistant professor/lecturer in Huazhong University of Science and Technology in China. Dr. Sheng holds five patents and has received several academic and industrial awards. He has co-authored five books and has published more than 100 research papers covering topics such as dynamics, noise and vibration, and tribology.

Shuguang Zuo, Ph.D.
Professor of Mechanical Engineering
Automobile Engineering School
Tongji University, Shanghai, P.R. China

Shuguang Zuo is a professor of automotive engineering at Tongji University in the Peoples Republic of China. He received his B.S. in Mechanical Design in 1990 from Hunan Agricultural University, and his M.S. in 1993 and his Ph.D. in 1996 in Automotive Engineering from Jilin Institute of Technology (now called Jilin University). From 1996 to 1998, Dr. Zuo was the Post-Doctoral Researcher in Aerospace Technology at the Post-Doctoral Exchange Center of Nanjing University of Aeronautics and Astronautics and the Chunlan Enterprise Post-Doctoral Exchange Center. Since 1998, Dr. Zuo has worked as an Associate Professor, Director of Automotive Teaching–Research Office, and Deputy Director of the Vehicle Research Laboratory in Shanghai Tiedao University; and a Professor and Ph.D. Supervisor in the School of Automobiles at Tongji University. His achievements include being an 11th Shanghai Shuguang Scholar, an Accreditation Expert of High New Technology Achievement Transfer Projects in the Science and Technology Commission of Shanghai Municipality, an Accreditation Expert of the National Natural Science Foundation, and an Accreditation Expert of the National 863 Program Project. Dr. Zuo has many years of extensive research and teaching experience in automotive engineering. He has led and participated in more than 20 research projects and has applied for four patents, two of which have already been granted. Dr. Zuo’s current major research projects include two projects for the National Natural Science Foundation, "Body Vibration and Noise Characteristic Research on Fuel Cell Electric Vehicle" and "Noise and Vibration Characteristics Experimental and Theoretical Analysis on Fuel Cell Electric Vehicle" sub-items of the National 863 Program Project, “Electric Vehicle–Fuel Cell Vehicle Complete Car Project.” He has published more than 40 papers in Chinese and foreign core academic journals and is a member of SAE International.